The Spanning Galaxy Problem

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Abstract

In a directed graph, an *star* is an out-branching with at least one arc, in which the root dominates all the other vertices. A *galaxy* is a vertex-disjoint union of stars. In this paper, we consider the Spanning Galaxy Problem of deciding whether a digraph D has a spanning galaxy or not. We show that although this problem is NP-complete (even when restricted to acyclic digraphs), it becomes polynomial-time solvable when restricted to strongly connected digraphs. We prove indeed that in the strongly connected case, the problem is equivalent to finding a strong subgraph with an even number of vertices. As a consequence of this work, we improve some results concerning the notion of directed star arboricity of a digraph D, which is the minimum number of galaxies needed to cover all the arcs of D. We show in particular that $dst(D) \leq \Delta(D) + 1$ for every digraph D and that $dst(D) \leq \Delta(D)$ for every acyclic digraph D.

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