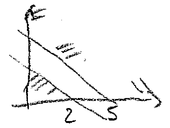
- $\max \; x_1 - x_2$

$$x_1 + x_2 \le 2$$
$$2x_1 + 2x_2 \ge 5$$
$$x_1, x_2 \ge 0$$



$\max \; x_1 - x_2$

$$x_1 + x_2 + x_3 = 2$$
$$2x_1 + 2x_2 - x_4 = 5$$
$$x_1, x_2, x_3, x_4 \ge 0$$

x_1	x_2	x_3	x_4	$-z$	b
1	1	1	0	0	2
2	2	0	-1	0	5
1	-1	0	0	1	0

we do not have a basic feasible sol.

Phase I AUXILIARY PROBLEM:

$$\max \; -x_5 \equiv \min x_5$$
$$x_1 + x_2 + x_3 = 2$$
$$2x_1 + 2x_2 - x_4 + x_5 = 5$$
$$x_1, x_2, x_3, x_4, x_5 \ge 0$$

x_1	x_2	x_3	x_4	x_5	$-z$	$-w$	b
1	1	1	0	0	0	0	2
2	2	0	-1	1	0	0	5
1	-1	0	0	0	1	0	0
0	0	0	0	-1	0	1	0

it is always easy from here to find a canonical form by letting x_5 enter the basis

	x_1	x_2	x_3	x_4	x_5	$-z$	$-w$	b	
x_3 leaves $\rightarrow$	$\boxed{1}$	1	1	0	0	0	0	2	(2/1)
	2	2	0	-1	1	0	0	5	(5/2)
	1	-1	0	0	0	1	0	0	
IV − 2 I	+2	+2	0	-1	0	0	1	+5	

$\uparrow$ enters

we have a basic feasible sol!

	1	1	1	0	0	0	0	2
II − 2I	0	0	-2	-1	1	0	0	1
III − I	0	-2	-1	0	0	0	0	-2
IV − 2I	0	0	-2	-1	0	0	0	+1

Keep $-z$ always in basis

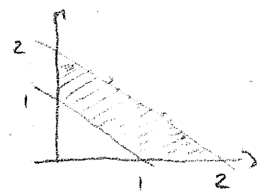
$\Rightarrow w^* = -1 \Rightarrow$ no sol with $x_5 = 0$ exists $\Rightarrow$ no feasible sol to initial problem

- $\max \; x_1 - x_2$

$$x_1 + x_2 \le 2$$
$$2x_1 + 2x_2 \ge 2$$
$$x_1, x_2 \ge 0$$

$\max \; x_1 - x_2$

$$x_1 + x_2 + x_3 = 2$$
$$2x_1 + 2x_2 - x_4 = 2$$
$$x_1, x_2, x_3, x_4 \ge 0$$



Phase I AUXILIARY PROBLEM:

$$\max \; -x_5 \equiv \min x_5$$
$$x_1 + x_2 + x_3 = 2$$
$$2x_1 + 2x_2 - x_4 + x_5 = 2$$
$$x_1, x_2, x_3, x_4, x_5 \ge 0$$

x_1	x_2	x_3	x_4	x_5	$-z$	$-w$	b
1	1	1	0	0	0	0	2
2	2	0	-1	1	0	0	2
1	-1	0	0	0	1	0	0
0	0	0	0	-1	0	1	0

canonical form with x_5 entering the basis

	x_1	x_2	x_3	x_4	x_5	$-z$	$-w$	b
	1	1	1	0	0	0	0	2
x_5 leaves $\rightarrow$	$\boxed{2}$	2	0	-1	1	0	0	2
	1	-1	0	0	0	1	0	0
	2	2	0	-1	0	0	1	2

x_1	x_2	x_3	x_4	x_5	$-z$	$-w$	d
0	0	1	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0	1
1	1	0	$-\frac{1}{2}$	$\frac{1}{2}$	0	0	1
0	-2	0	$+\frac{1}{2}$	$-\frac{1}{2}$	1	0	-1
0	0	0	0	-1	0	0	0

$\Rightarrow w^* = 0 \Rightarrow x_5 = 0 \Rightarrow$ we have a starting feasible sol for initial problem

Phase II

We keep only what we need:

x_1	x_2	x_3	x_4	$-z$	b
0	0	1	$\boxed{\frac{1}{2}}$	0	1
1	1	0	$-\frac{1}{2}$	0	1
0	-2	0	$\frac{1}{2}$	1	-1

we have a canonical form

0	0	2	1	0	2
1	1	1	0	0	2
0	-2	-1	0	1	-2

Optimal sol $\begin{bmatrix} x_1 = 2 \\ x_2 = 0 \\ x_3 = 0 \\ x_4 = 2 \end{bmatrix}$, $z = 2$

Note that finding a feasible is in general computationally as difficult as finding an optimal solution

In dictionary form:

$$\max\ x_1 - x_2$$
$$x_1 + x_2 \le 2$$
$$2x_1 + 2x_2 \ge 5$$
$$x_1, x_2 \ge 0$$

$$\max\ -x_0$$
$$x_1 + x_2 - x_0 \le 2$$
$$-2x_1 - 2x_2 - x_0 \le -5$$
$$x_0, x_1, x_2 \ge 0$$

$$x_3 = 2 - x_1 - x_2 + x_0$$
$$x_4 = -5 + 2x_1 + 2x_2 + x_0$$
$$\overline{}$$
$$w = -x_0$$

it is infeasible because $x_4 = -5 \not\geq 0$
but it can be made feasible by letting x_0 enter the basis

which var should leave?
The most infeasible: the var with the b term whose negative value has the largest magnitude

$$x_0 = 5 - 2x_1 - 2x_2 + x_4$$
$$x_3 = 2 - x_1 - x_2 + 5 - 2x_1 - 2x_2 + x_4 = 7 - 3x_1 - 3x_2 + x_4$$
$$w = -5 + 2x_1 + 2x_2 - x_4$$

$x_1 \le \frac{5}{2} = 2.5$

$x_1 \le \frac{7}{3} = 2.3$

x_1 enters x_3 exists

$\vdots$