Kempe Chains and Rooted Minors

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A (minimal) *transversal* of a partition is a set which contains exactly one member from each member of the partition and nothing else. We study the following problem. Given a transversal T of a proper coloring $\mathfrak{C}$ of some graph G, is there a partition $\mathfrak{H}$ of a subset of $V(G)$ into connected sets such that T is a transversal of $\mathfrak{H}$ and such that two sets of $\mathfrak{H}$ are adjacent if their corresponding vertices from T are connected by a path using only two colors?

It has been conjectured earlier that for any transversal T of a coloring $\mathfrak{C}$ of order k of some graph G such that any pair of color classes induces a connected graph, there exists a partition $\mathfrak{H}$ into pairwise disjoint pairwise adjacent sets such that T is a transversal of $\mathfrak{H}$ (which would prove Hadwigers conjecture for the class of uniquely colorable graphs); this is open for each $k \geq 5$ and follows from our results for the case that $k = 5$ and the subgraph induced by T is connected.