

DM19 – Algorithms and Complexity – E03 – Lecture 14

Announcement - repeat

The Friday discussion sections have been cancelled. There will only be discussion sections on Thursdays at 10:15 in U2.

Lecture, December 1

We finished branch-and-bound from the second set of notes and began on heuristics from the first set of notes, covering up through and including section 10.4.2 on genetic algorithms

Lecture, December 8

We will finish heuristics from the first set of notes and begin on on-line algorithms from: <http://www.uni-paderborn.de/fachbereich/AG/agmadh/WWW/english/scripts.html#Albers>

These are notes written by Susanne Albers and can be accessed through the Web page for this course.

Lecture, December 15

We will continue with on-line algorithms.

Problems to be discussed in week 51

- Design a genetic algorithm for the MAX-SAT problem.

- Show that LFU is not competitive for any $k \geq 3$. This means that for any function $f(k)$ and any constant a , there is a request sequence on which LFU has C_{LFU} faults, while an optimal off-line algorithm has C_{OPT} faults, and $C_{LFU} > f(k) \cdot C_{OPT} + a$. LFU is defined as follows: For each page in the system, LFU keeps a counter keeping track of how many times the page is requested. When it is necessary to evict page, the page chosen has the smallest counter value. (If this smallest value is not unique, any of the pages with that smallest value may be chosen.)

- Prove Thm 1 (on page 3 of the notes on on-line algorithms) for FIFO. (The proof given in the notes is only for LRU.)

Note: The next 2 problems are from the textbook *Online Computation and Competitive Analysis*, by Allan Borodin and Ran El-Yaniv.

- If a deterministic algorithm for paging is c -competitive, then $c \geq k$, even if there are only $k + 1$ pages in the system, where the fast memory can contain k pages. Prove that MARKING is H_k -competitive, when there are only $k + 1$ pages in the system.
- Show that in general, MARKING is not H_k -competitive. (Hint: It is sufficient to consider the case $k = 2$, with $N = 4$ pages total.)
- Consider the on-line bin-packing problem: There is a large supply of bins, all of capacity 1. The request sequence is a sequence of objects $\langle r_1, \dots, r_m \rangle$. Object r_i has size s_i , which is known when the object arrives. The goal is put each object in some bin, so that no bin contains objects whose sizes add to more than 1, using as few bins as possible. The algorithm First-Fit places an object in the first bin in which it fits. Show that First-Fit is 2-competitive. (Actually, it is 17/10-competitive, but that's much harder to show.)
- Show that the First-Fit algorithm for bin-packing cannot be better than 5/3-competitive. (Hint: use items of size $\frac{1}{7} + \epsilon$, $\frac{1}{3} + \epsilon$, and $\frac{1}{2} + \epsilon$, where ϵ is very small.)