Institut for Matematik og Datalogi
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March 7, 2003
JFB

On-Line Algorithms – F03 – Note 6

Lecture, March 7

Kim Skak Larsen covered through section 4.3 of chapter 4.

Lecture, March 14

Kim Skak Larsen will cover section 4.4. We will also finish chapter 6.

Lecture, March 21

We will cover chapter 7 quickly and cover much of chapter 8.

Problems for Wednesday, March 19

1. (Part of Exercise 6.4.) Show that the algorithm PERM_π is neither a marking algorithm nor a conservative algorithm. Try using $N = k + 2$.

2. Do Exercise 6.5.

3. Do Exercise 6.6.

4. Consider the Dual Bin Packing Problem from the first weekly note, and assume we are only considering fair algorithms. Consider the following adversary against a deterministic algorithm $\mathbb{A}$: Give $\mathbb{A}$ the following request sequence, divided into three phases. Phase 1 consists of n small items of size $\frac{1}{n}$. Phase 2 consists of items, one for each bin which $\mathbb{A}$ did not fill completely with size equal to the empty space in that bin, sorted in decreasing order. After these are given, $\mathbb{A}$ has filled all bins completely and so must reject the items in Phase 3, which consists of $\frac{n^2}{4}$ items of size $\frac{1}{n}$.

To analyze this, let q denote the number of bins in $\mathbb{A}$'s configuration which have at least 2 items after the first phase.

In the case where $q < \frac{n}{4}$, we know that $\mathbb{A}$ has at least $n - q \geq \frac{3n}{4}$ bins with at most one item after Phase 1. OPT can arrange the items from Phase 1 such that half of the bins contain two items and half contain no items.

In the case where $q \geq \frac{n}{4}$, we know that $\mathbb{A}$ has at least $\frac{n}{4}$ empty bins after Phase 1. OPT places each of the items from Phase 1 in a different bin.

a. Use the adversary above to show that no fair algorithm for the dual bin packing problem is more than $\frac{8}{6+n}$-competitive.

b. Try changing the above adversary to an adaptive on-line adversary. What result can you get?

c. Define an algorithm which uses a mixed strategy to solve the dual bin packing problem.

d. Define an algorithm which uses a behavioral strategy to solve the dual bin packing problem.