Institut for Matematik og Datalogi
Syddansk Universitet

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JFB

On-Line Algorithms – F05 – Lecture 7

Lecture, March 15

We finished chapter 6.

Lecture, March 29

We will begin looking at the article "The relative worst order ratio applied to paging", at http://www.imada.sdu.dk/∼joan/online/paging.pdf. In section 2, we will initially only consider definitions 1 and 2 and skip the others. Next we will cover Lemmas 6 and 7 and Theorem 5, followed by Lemmas 3, 4, and 5 and Theorem 4. Then we will cover section 5.

Lecture, April 5

We will continue looking at "The relative worst order ratio applied to paging".

Problems for April 4

1. Show that with the relative worst order ratio, for a given problem, the ordering as to which algorithms are better than which is transitive. Show that if $\mathrm{WR}_{\mathbb{A},\mathbb{B}} \geq 1$ and $\mathrm{WR}_{\mathbb{B},\mathbb{C}} \geq 1$, then $\mathrm{WR}_{\mathbb{A},\mathbb{C}} \geq \mathrm{WR}_{\mathbb{B},\mathbb{C}}$. Furthermore, show that if $\mathrm{WR}_{\mathbb{A},\mathbb{B}}$ is bounded above by some constant, then $\mathrm{WR}_{\mathbb{A},\mathbb{C}} \geq \mathrm{WR}_{\mathbb{A},\mathbb{B}}$.

2. Lemma 4 in the article "The relative worst order ratio applied to paging" does not hold if the conservative algorithm is allowed look-ahead. How do you know this? Where does the proof fail?

3. Find another sequence which would separate LRU's and FWF's behavior under the relative worst order ratio. (It's not necessary to get as large a ratio as the one in the article. Try for $\frac{3}{2}$.)

4. Try defining an algorithm which is based on FIFO and uses look-ahead. What is its relative worst order ratio compared to FIFO? To LRU?

5. Consider the algorithm for dual bin packing (fixed number of bins, maximizing the number of accepted items) behaves exactly as First-Fit would unless the item x is larger than $\frac{1}{2}$ and would be placed in the l ast bin, bin n. The algorithm FF_n rejects such an item and is thus not fair.

   Show that FF_n is better than FF, according to the relative worst order ratio.