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JFB

On-Line Algorithms – F06 – Lecture 9

Lecture, March 22

We began looking at the article "The relative worst order ratio applied to paging", at http://www.imada.sdu.dk/~joan/online/paging2.pdf. (See the course's homepage.) In section 2, we will initially only consider definitions 1 and 2 and skip the others. Next we will cover up through Theorem 5 of section 4, and then section 6. RLRU was briefly introduced.

Lecture, March 29

We will cover sections 3 and 5, plus the definitions for relatedness and weakly comparable in section 2 of "The relative worst order ratio applied to paging".

Lecture, April 5

We will finish section 6 and cover section 7 of "The relative worst order ratio applied to paging", and then chapter 7 in the textbook quickly.

Problems for March 28

1. Show that with the relative worst order ratio, for a given problem, the ordering as to which algorithms are better than which is transitive. Show that if $\mathrm{WR}_{\mathbb{A},\mathbb{B}} \geq 1$ and $\mathrm{WR}_{\mathbb{B},\mathbb{C}} \geq 1$, then $\mathrm{WR}_{\mathbb{A},\mathbb{C}} \geq \mathrm{WR}_{\mathbb{B},\mathbb{C}}$. Furthermore, show that if $\mathrm{WR}_{\mathbb{A},\mathbb{B}}$ is bounded above by some constant, then $\mathrm{WR}_{\mathbb{A},\mathbb{C}} \geq \mathrm{WR}_{\mathbb{A},\mathbb{B}}$.

2. Lemma 4 in the article "The relative worst order ratio applied to paging" does not hold if the conservative algorithm is allowed look-ahead. How do you know this? Where does the proof fail?

3. Find another sequence which would separate LRU's and FWF's behavior under the relative worst order ratio. (It's not necessary to get as large a ratio as the one in the article. Try for $\frac{3}{2}$.)
4. Try defining an algorithm which is based on FIFO and uses look-ahead. What is its relative worst order ratio compared to FIFO? To LRU?
5. Consider the algorithm for dual bin packing (fixed number of bins, maximizing the number of accepted items) behaves exactly as First-Fit would unless the item x is larger than $\frac{1}{2}$ and would be placed in the l ast bin, bin n. The algorithm FF_n rejects such an item and is thus not fair.

   Show that FF_n is better than FF, according to the relative worst order ratio.

Problems for April 4

1. In the definition of RLRU, in the case where p is requested, but there is not fault, the algorithm only marks the next page if it is different from the previous. What happens to the results on RLRU if this check is removed and the page is always marked. What if it is never marked?
2. In the definition of RLRU, what if you change the condition starting a new phase to be "the $k + 2$nd different page since the start of the last phase was found" or "this was the $k + 2$nd fault since the start of the last phase"? What happens to the results on RLRU?
3. Compare MTF and TRANS for the list processing problem, using the relative worst order ratio.
4. How would you define a "strict relative worst order ratio"?
5. Work out an example showing how to change a worst case ordering for LRU to a worst case ordering for PERM_π.