

## On-Line Algorithms – F14 – Lecture 10

### Lecture, March 20

We covered section 10.1 on March 17 and section 10.4 of chapter 10 on March 20. We covered up through Theorem 12.2 in chapter 12 in the textbook.

### Lecture, March 24

We will finish section 12.2.2 in the textbook.

### Problems for March 24, possibly continuing on March 26

1. Exercise 10.1 in the textbook.
2. (Easy) Show that the makespan problem for identical machines is NP-hard.
3. Suppose that GREEDY is allowed  $n$  identical machines, while OPT is only allowed to use  $m < n$  machines. Give a sequence showing that the ratio of GREEDY's performance to OPT's can be at least  $1 + \frac{m-1}{n}$  for the makespan problem. Then show that GREEDY can always achieve this ratio against such a bounded OPT.
4. Consider remark 12.1 on page 208. What is meant here? Why is there no problem if the loads can be greater than 1? (Do not try to prove the desired result for loads of at most 1.)
5. Define POST-GREEDY with release dates as the algorithm which assigns a new job (given at its release date) to the first processor which becomes free. (Jobs have processing times which may be unknown, and only one job may be running on a processor at a time. There are  $m$  processors.) Show that POST-GREEDY is  $(2 - \frac{1}{m})$ -competitive.

6. Consider the algorithm for dual bin packing (fixed number of bins, maximizing the number of accepted items) behaves exactly as First-Fit would unless the item  $x$  is larger than  $\frac{1}{2}$  and would be placed in the last bin, bin  $n$ . The algorithm  $\text{FF}_n$  rejects such an item and is thus not fair.

Show that  $\text{FF}_n$  is better than  $\text{FF}$ , according to the relative worst order ratio.