Institut for Matematik og Datalogi
Syddansk Universitet

May 11, 2017
JFB

Online Algorithms – F17 – Assignment 3

Assignment due Friday, May 26, 12:15

Note that this is part of your exam project, so it must be approved in order for you to take the exam in June, and you may not work with others not in your group. If it is late, it will not be accepted (though it could become an assignment you redo). You may work in groups of two (or three). Turn in the assignment through the SDU Assignment system in Blackboard, and remember to keep your receipt. Turn in one PDF file per group.

1. Consider the lower bound proof in Theorem 1 of the article "Online Bin Packing with Advice", with $n = 6$ and $k = 2$. Which sequences are produced, and how much advice is needed? Explain.

2. Go through the reduction from 2-SGKH to Binary Separation with the string $\langle 1, 1, 1, 0 \rangle$ as input. Suppose you want to produce values in the range $[1/2, 3/4]$. You may assume that your algorithm for guessing "large" or "small" for Binary Separation alternates, starting with "large".

3. Consider the following advice for the makespan restricted machines scheduling problem on N identical machines: One bit of advice (in total, not per request) indicating whether the first machine among the allowable machines should always be used, or the last, when there is a choice.

   (a) Define an algorithm to use this advice effectively (specify what the algorithm should do with jobs with unit load, but different allowable sets of machines).

   (b) Following the proof of the lower bound in Theorem 12.3, what sequence would be produced by letting ALG be the GREEDY algorithm? (Define this sequence generally, based on the number of

machines, N, and assume that GREEDY chooses the lower numbered machine to break ties.) Show the effect this advice defined above would have on this sequence. How would your algorithm schedule this sequence, and what would the performance ratio be, compared to OPT? (Note that if your algorithm does not do better on this sequence than GREEDY would, you should redefine your algorithm.)

(c) Give an example input sequence where all loads are 1, showing that the performance ratio compared to OPT is still at least 2.

(d) Give an example family of input sequences showing that the performance ratio compared to OPT is at least $1 + \log_4 N$, for N a power of 2.

4. Consider the makespan restricted machines scheduling problem on N identical machines. Present an online algorithm with advice which achieves optimality. How many bits of advice do you use?

5. Explain the proof that Dominating Set is AOC-hard in your own words. Also give an example.

6. Find an $(8, 6, 2)$-covering design of size 3. Show how to use this covering design for the following instance of of MinASG: $\langle 0, 1, 0, 0, 1, 0, 0, 0 \rangle$.

7. How would you define the online Shortest Path problem, following the definitions of Vertex Cover and Cycle Finding, for example. Show that Shortest Path is in AOC if all weights are 1.