Department of Mathematics and Computer Science
University of Southern Denmark, Odense

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Marco Chiarandini

# DM828 - Introduction to Artificial Intelligence

## Exercise Sheet 2, Autumn 2011 [pdf format]

Prepare the following exercises for discussion in class on Thursday, November 17.

## Exercises

1. Given the full joint distribution in Figure 13.3 of the course book, calculate the following:
   - $P(toothache)$
   - $P(Cavity)$
   - $P(Toothache|cavity)$
   - $P(Cavity|toothache \vee catch)$
2. Show that the statement
   $$P(A, B|C) = P(A|C)P(B|C)$$
   is equivalent to either of the statements
   $$P(A|B, C) = P(A|C) \qquad \text{and} \qquad P(B|A, C) = P(B|C)$$
3. Often we need to carry out reasoning over some pair of variables $X$, $Y$ conditioned on the value of other variable $E$.
   a Using the definitions of conditional probabilities, prove the conditionalized version of the product rule:
   $$P(x, y|e) = P(x|y, e)P(y|e)$$
   b Prove the conditionalized version of Bayes' rule:
   $$P(y|x, e) = P(x|y, e)P(y|e)/P(x|e)$$
4. Following the example on medical diagnosis of Section 13.5.1 of the text book, give the probability $P(M|s)$ by completing the normalization calculation. Assume 0.05 as a reasonable value for $P(s|\neg m)$ and ignore the value for $P(s)$.
5. Three prisoners $A$, $B$, $C$ are in their cells. They are told that one of them will be executed the next day and the others will be pardoned. Only the gorvenor knows who will be executed. Prisoner $A$ asks the guard a favor. "Please ask the governor who will be executed, and then tell either prisoner $B$ or $C$ that they will be pardoned." The guard does as was asked and then comes back and tells prisoner $A$ that he has told prisoner $B$ that he ($B$) will be pardoned. What are prisoner $A$'s chances of being executed, given this message? Is there more information

than before his request to the guard? At least two approaches are possible to answer this questions: classical probability and via Bayes' rule. Develop them both. [This problem adapted from Pearl 1988, is also a variant of another rather famous puzzle: you are given the choice of three doors. Behind one door is a car; behind the others, goats. They are placed randomly. After you have chosen a door, the door remains closed. Another person, who knows what is behind the doors, now has to open one of the two remaining doors, and the door he opens must have a goat behind it. If both remaining doors have goats behind them, he chooses one randomly. You are now posed with the question, whether you want to stay with your first choice or to switch to the last remaining door. ]

6. Suppose that you go out to purchase an automobile. The probability that you will go to dealer 1, $d_1$ is 0.2. The probability of going to dealer 2, $d_2$ is 0.4. There are only three dealers you are considering and the probability that you go to the third, $d_3$ is 0.4. At $d_1$ the probability of purchasing a particular automobile, $a_1$, is 0.2.; at dealer $d_2$ the probability of purchasing a automobile $a_1$ is 0.4. Finally, at dealer $d_3$, the probability of purchasing $a_1$ is 0.3. Suppose you purchase automobile $a_1$. What is the probability that you purchased it at dealer $d_2$?

7. Exercise 14.5 of the text book.

8. Exercise 14.12 of the text book.