Department of Mathematics and Computer Science
University of Southern Denmark, Odense

February 10, 2016
Marco Chiarandini

# **FF505** – Computational Science

Week 6, Spring 2016

---

## MATLAB Exercises

Work in small groups at the computer to solve the following exercises.

1. Read chapter 9 from the course reference [SW] and finish the exercise from the previous week on population dynamics.

2. Create a new function in a file named `addme.m` that accepts one or two inputs and computes the sum of the number with itself or the sum of the two numbers. The function must be then able to return one or two outputs (a result and its absolute value). [Hint: you may need the system variable `nargin`]

3. Curve fitting. Read chapter 12 from the course reference [SW]. Then generate 10 points in the Cartesian plane and fitt a polynomial of order three on those points. Compare the quality of two alternative solution one via `ployfit` and another via `fminsearch`. Finally plot the points and the fitted curve.

4. Projectile trajectory. From classical physics we know that the position vector $\boldsymbol{p}$ of a projectile in the 3D space is described as follows:

$$\boldsymbol{p}_t = \boldsymbol{p}_0 + \boldsymbol{u}_t s_m t + \frac{g t^2}{2}$$

where $s_m$ is the muzzle velocity (speed at which the projectile left the weapon), $\boldsymbol{u}_t$ is the direction the weapon was fired, and $g_y = -9.81 \text{ms}^{-1}$ is the gravity.

- Predict the landing spot.
- Plot in a 3D plot the speed vector field of the trajectory.
- Given a firing point $\boldsymbol{S}$ and $s_m$ and a target point $\boldsymbol{E}$, we want to know the firing direction $\boldsymbol{u}$, $|\boldsymbol{u}| = 1$.

$$\begin{aligned}
E_x &= S_x + u_x s_m t_i + \frac{1}{2} g_x t_i^2 \\
E_y &= S_y + u_y s_m t_i + \frac{1}{2} g_y t_i^2 \\
E_z &= S_z + u_z s_m t_i + \frac{1}{2} g_z t_i^2 \\
1 &= u_x^2 + u_y^2 + u_z^2
\end{aligned}$$

four eq. in four unknowns, leads to:

$$|\boldsymbol{g}|^2 t_i^4 - 4(\boldsymbol{g} \cdot \boldsymbol{\Delta} + s_m^2) t_i^2 + 4|\boldsymbol{\Delta}|^2 = 0, \qquad \boldsymbol{\Delta} = \boldsymbol{E} - \boldsymbol{S}$$

solve in $t$, and interpret the solution.

- Let's introduce drag and air resistance. In a highly simplified model the drag force can be described as: $D = -kv - cv^2$, where $v$ is the velocity of the projectile and $k$ and $c$ are parameters. The equation of motion is a non-linear differential equation:

$$\boldsymbol{p}_t'' = g - k\boldsymbol{p}_t' - c\boldsymbol{p}_t'|\boldsymbol{p}_t'|$$

  It can be solved by iterative method via simulation and you may try doing that in Matlab. Alternatively, removing the second term we can solve

$$\boldsymbol{p}_t = \frac{\boldsymbol{g}t - \boldsymbol{A}e^{-kt}}{k} + \boldsymbol{B}, \quad \boldsymbol{A} = s_m\boldsymbol{u} - \frac{\boldsymbol{g}}{k}, \quad \boldsymbol{B} = \boldsymbol{p}_0 - \frac{\boldsymbol{A}}{k}.$$

  Is this still a parabola? Plot the trajectory.

- We wish to solve the motion equation in the simplified case with drag shown above and find the firing direction. In the case without second term you can still solve the system analytically. However, here you are asked to implement the following iterative targeting technique.
  - start with a tentative direction
  - simulate real projectile motion by a physics system
  - continue guessing until within a radius from target

  To continue guessing you shall use binary search: find a tentative upper or lower bound, then the opposite bound and continue by selecting as the next guess the average value between the lower and upper bound.