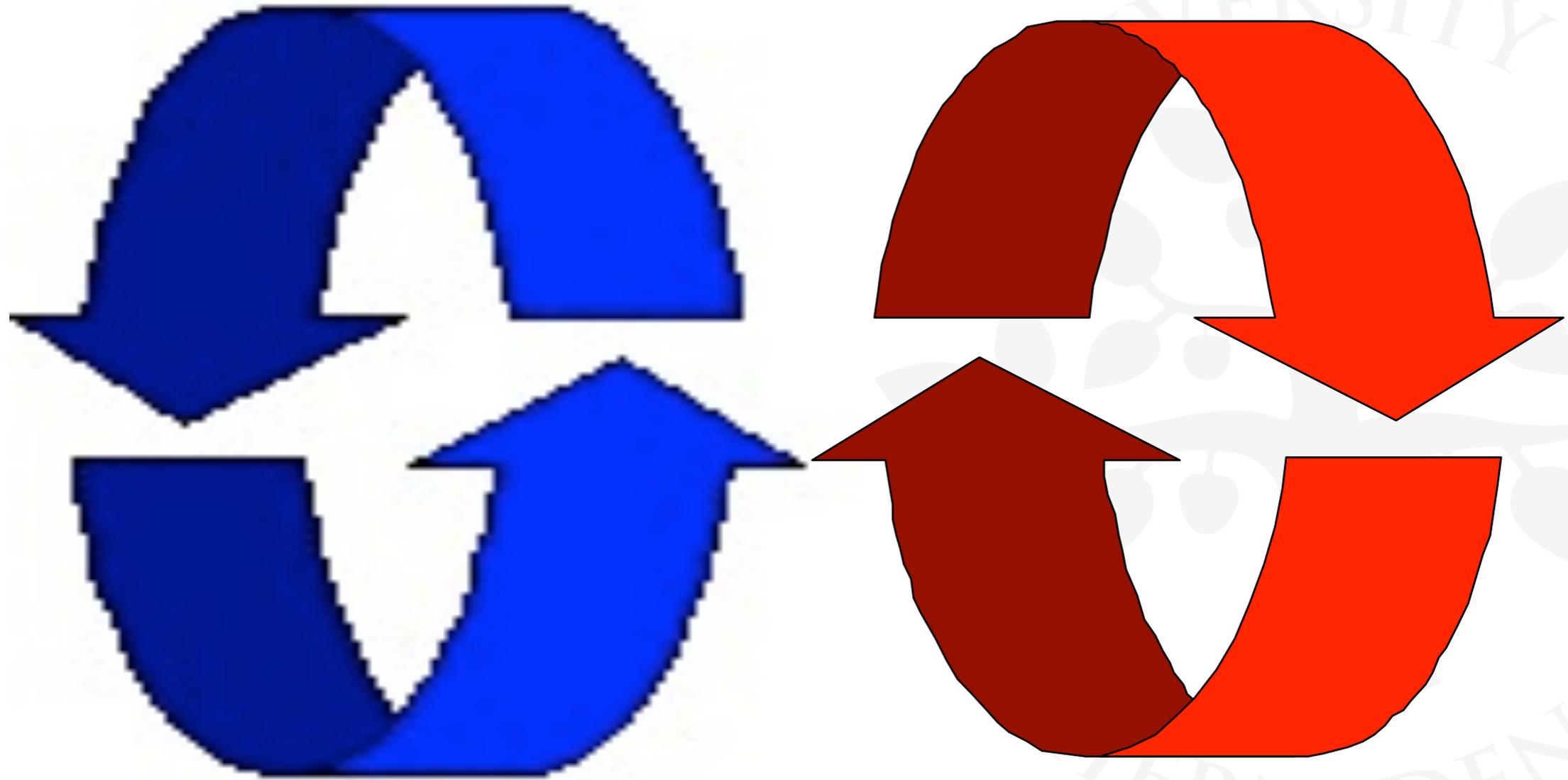


Chapter 3 Concurrent Execution



Repetition (Concepts, Models, and Practice)



UNIVERSITY OF SOUTHERN DENMARK



Repetition (Concepts, Models, and Practice)



◆ Concepts:





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- We adopt a **model-based approach** for the design and construction of concurrent programs





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- We use finite state models to represent concurrent behaviour
(Finite State Processes and Labelled Transition Systems)

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(**Finite State Processes and Labelled Transition Systems**)

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 - ◆ Safe model => safe program

◆ Models:

- We use finite state models to represent concurrent behaviour
(**Finite State Processes and Labelled Transition Systems**)

◆ Practice:

- We use **Java** for constructing concurrent programs

Repetition (Models; LTS, FSP)



Repetition (Models; LTS, FSP)



Model = simplified representation of the real world

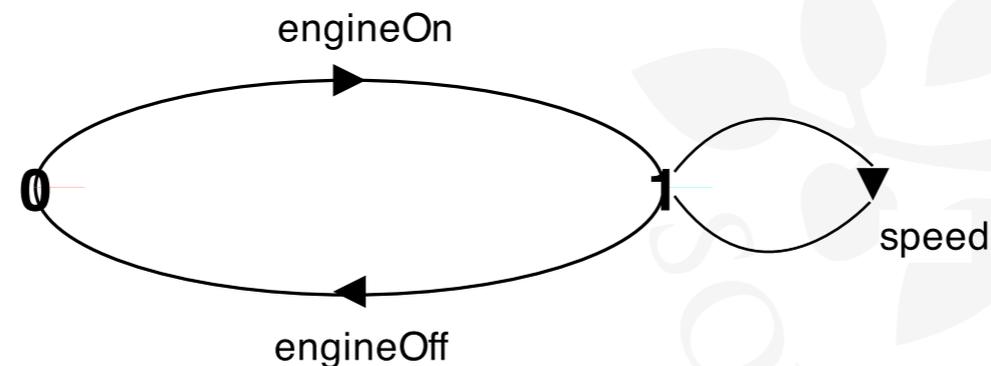


Repetition (Models; LTS, FSP)

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◆ Based on Labelled Transition Systems (**LTS**):

Focuses on **concurrency** aspects (of the program)
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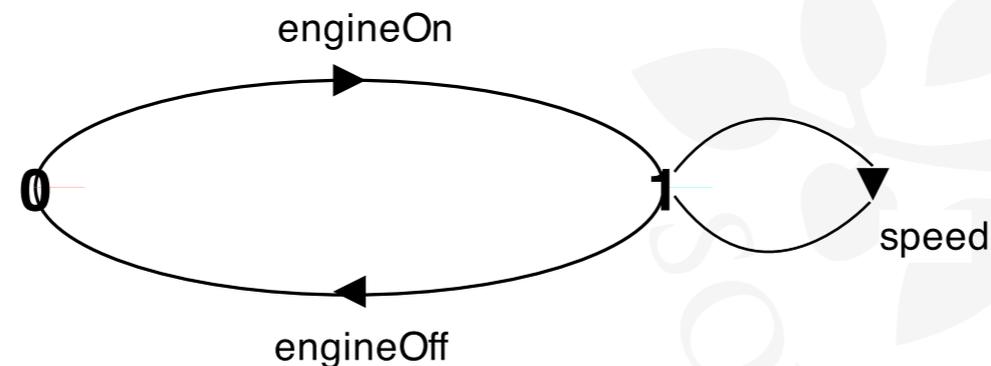


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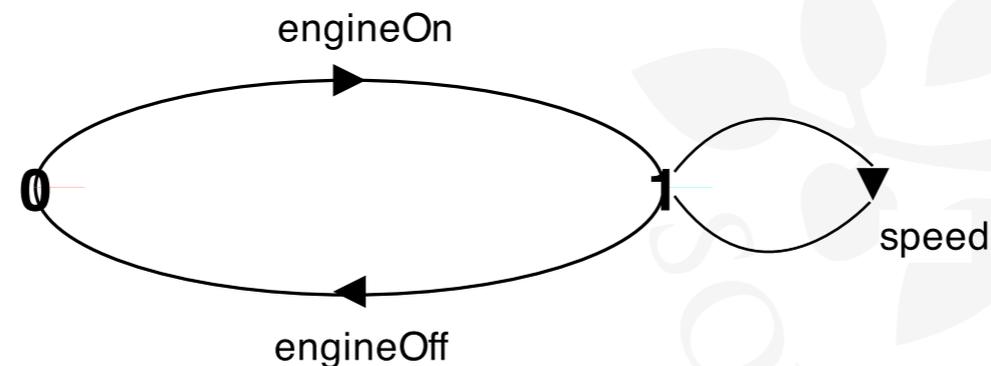


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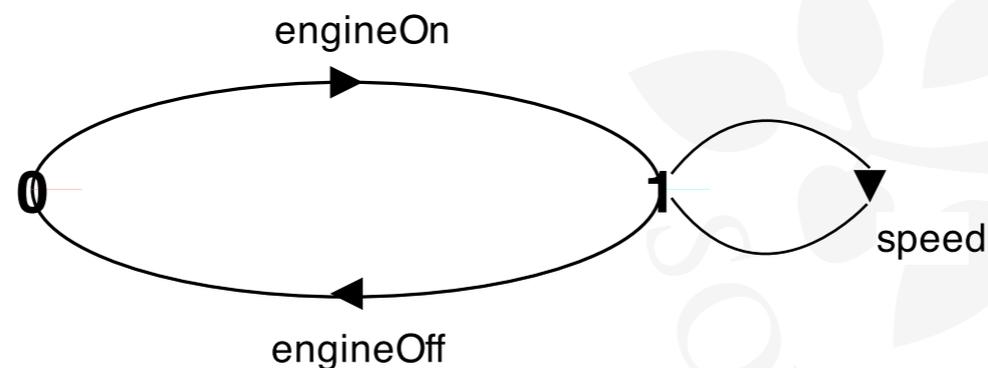
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(FSP):

```
EngineOff = (engineOn -> EngineOn) ,  
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```

Repetition (Finite State Processes; FSP)

Finite State Processes (FSP):

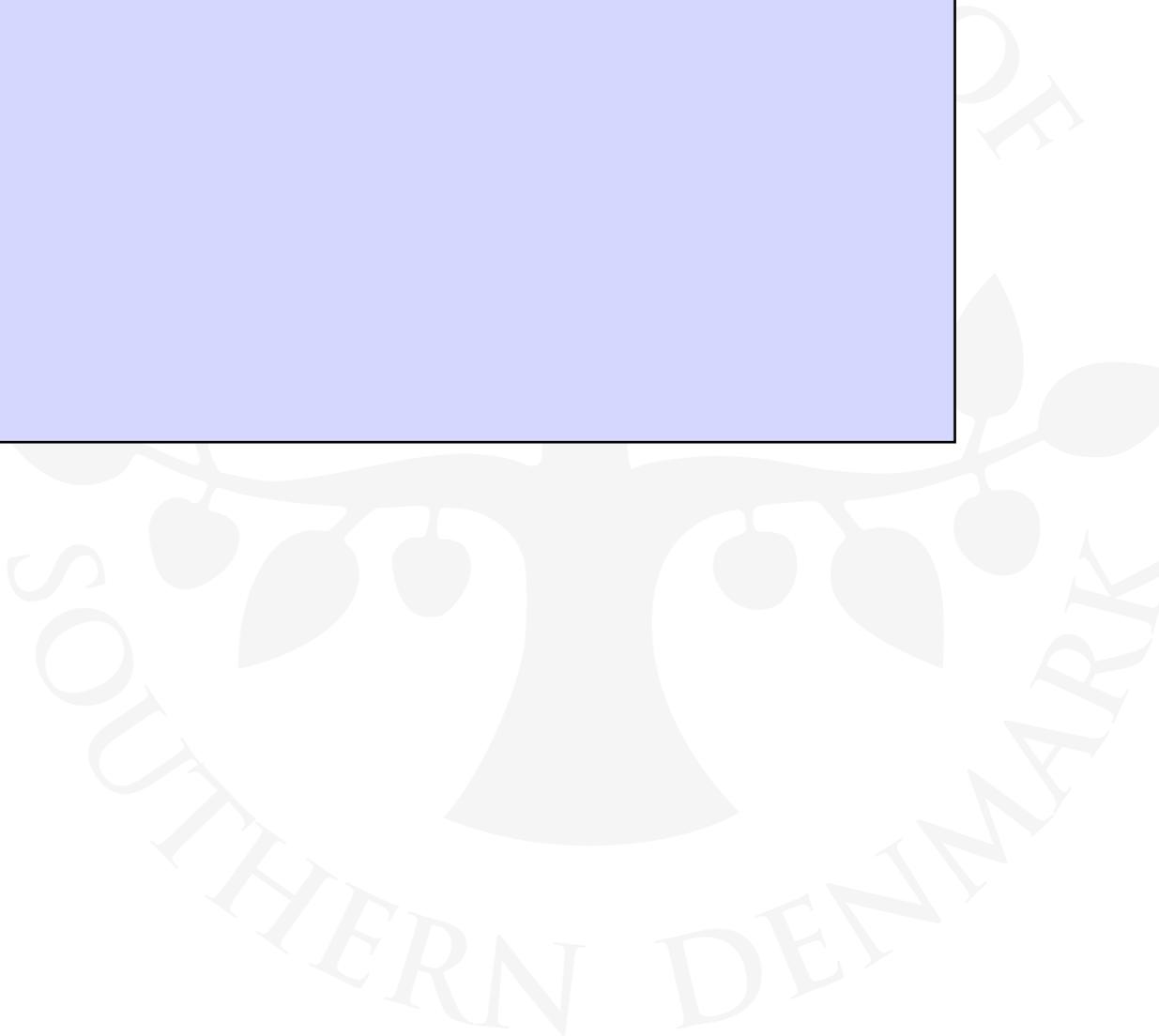




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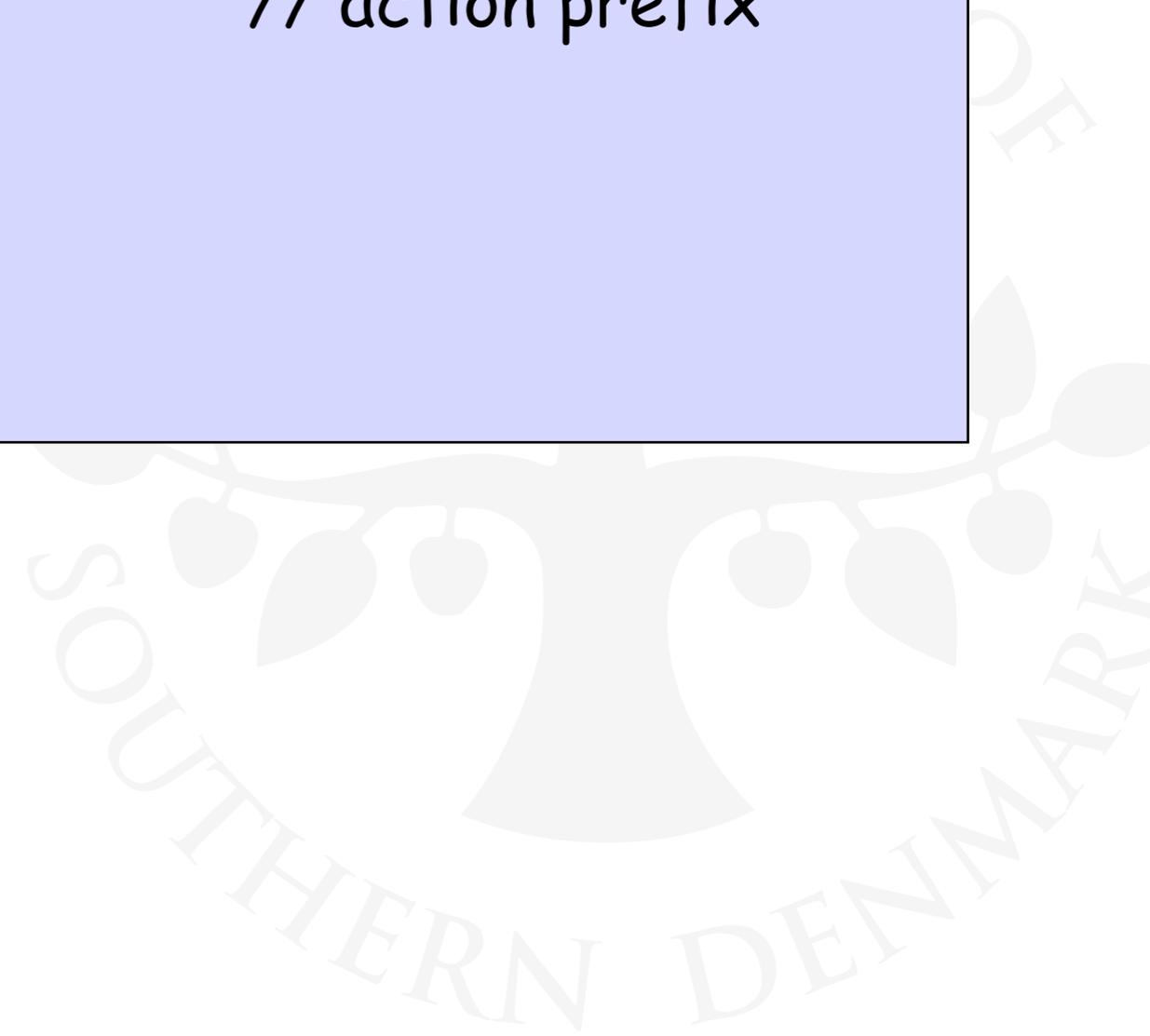




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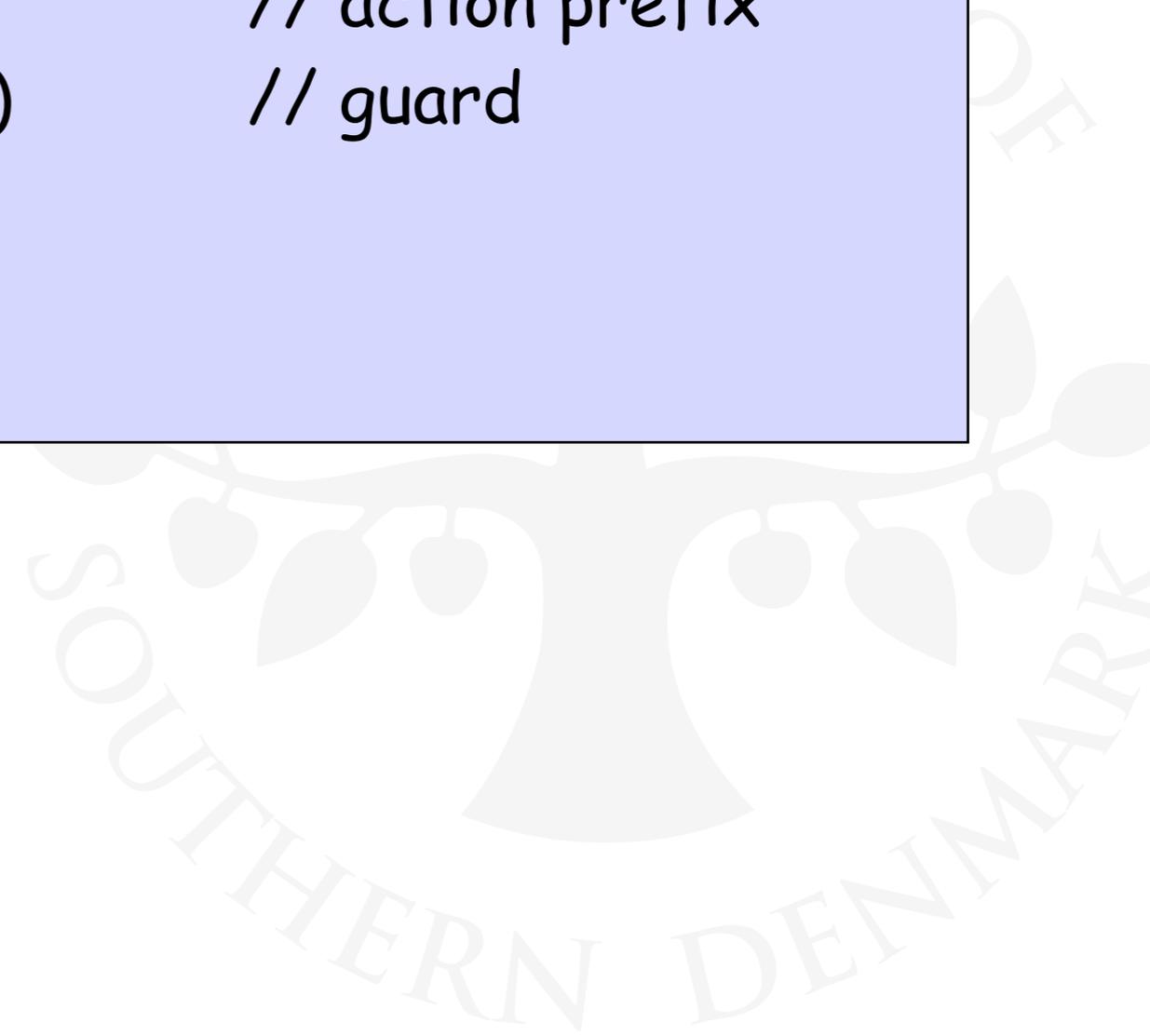




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◆ action indexing

$x[i:1..N] \rightarrow P$ or $x[i] \rightarrow P$



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$P(N=3) = \dots$



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- ◆ constant definitions

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const N = 3



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**Which constructions do not add expressive power?
(and are thus only "syntactic sugar").**



Repetition (Java Threads)

Subclassing `java.lang.Thread`:

Implementing `java.lang.Runnable`:





Repetition (Java Threads)

Subclassing `java.lang.Thread`:

```
class MyThread extends Thread {  
    public void run() {  
        // ...  
    }  
}
```

```
Thread t = new MyThread();  
t.start();  
// ...
```

Implementing `java.lang.Runnable`:



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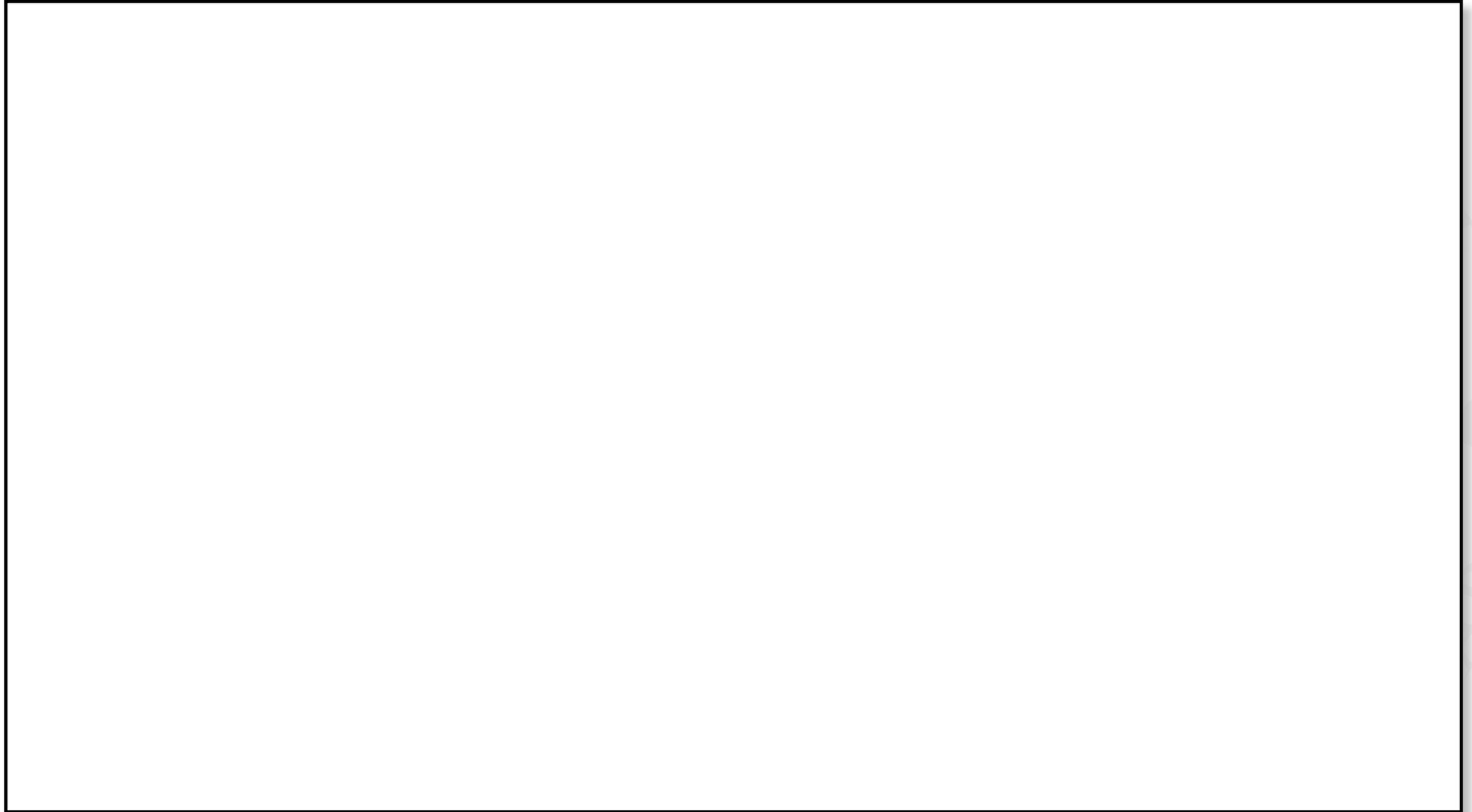
```
Thread t = new MyThread();  
t.start();  
// ...
```

Implementing `java.lang.Runnable`:

```
class MyRun implements Runnable {  
    public void run() {  
        // ...  
    }  
}
```

```
Thread t = new Thread(new MyRun());  
t.start();  
// ...
```

Chapter 3: Concurrent Execution





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Concepts: processes - concurrent execution
and interleaving



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process interaction



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Concepts: processes - concurrent execution
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Models: parallel composition of asynchronous processes
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process labelling, and action relabelling and hiding



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Concepts: processes - concurrent execution
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Practice: Multithreaded Java programs

Definition: Parallelism





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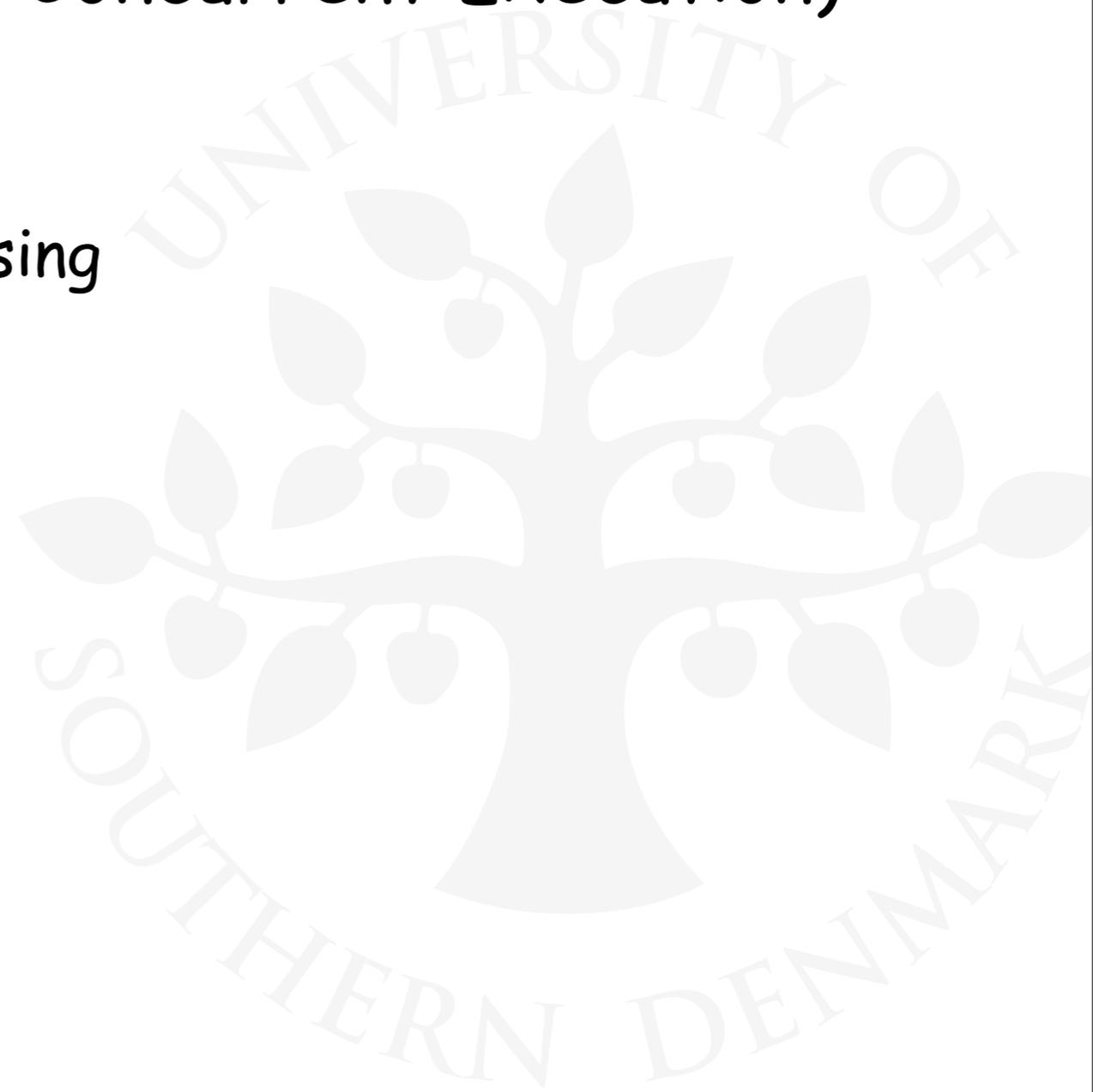
- ◆ **Parallelism** (aka. Real/True Concurrent Execution)





Definition: Parallelism

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 - **Physically simultaneous processing**

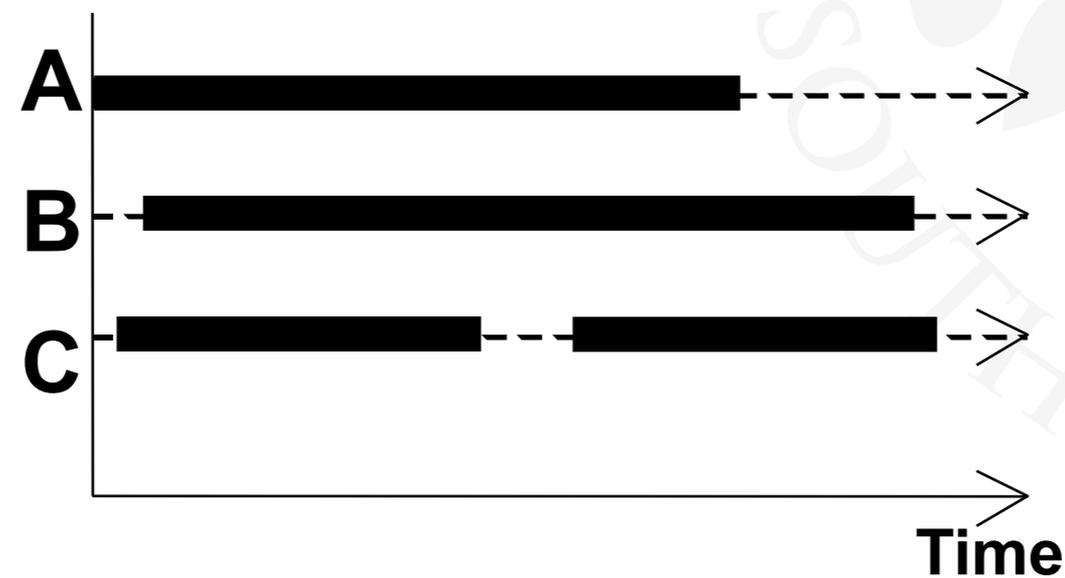




Definition: Parallelism

◆ Parallelism (aka. Real/True Concurrent Execution)

- Physically simultaneous processing
 - ◆ Involves multiple processing elements (PEs) and/or independent device operations



Definition: Concurrency



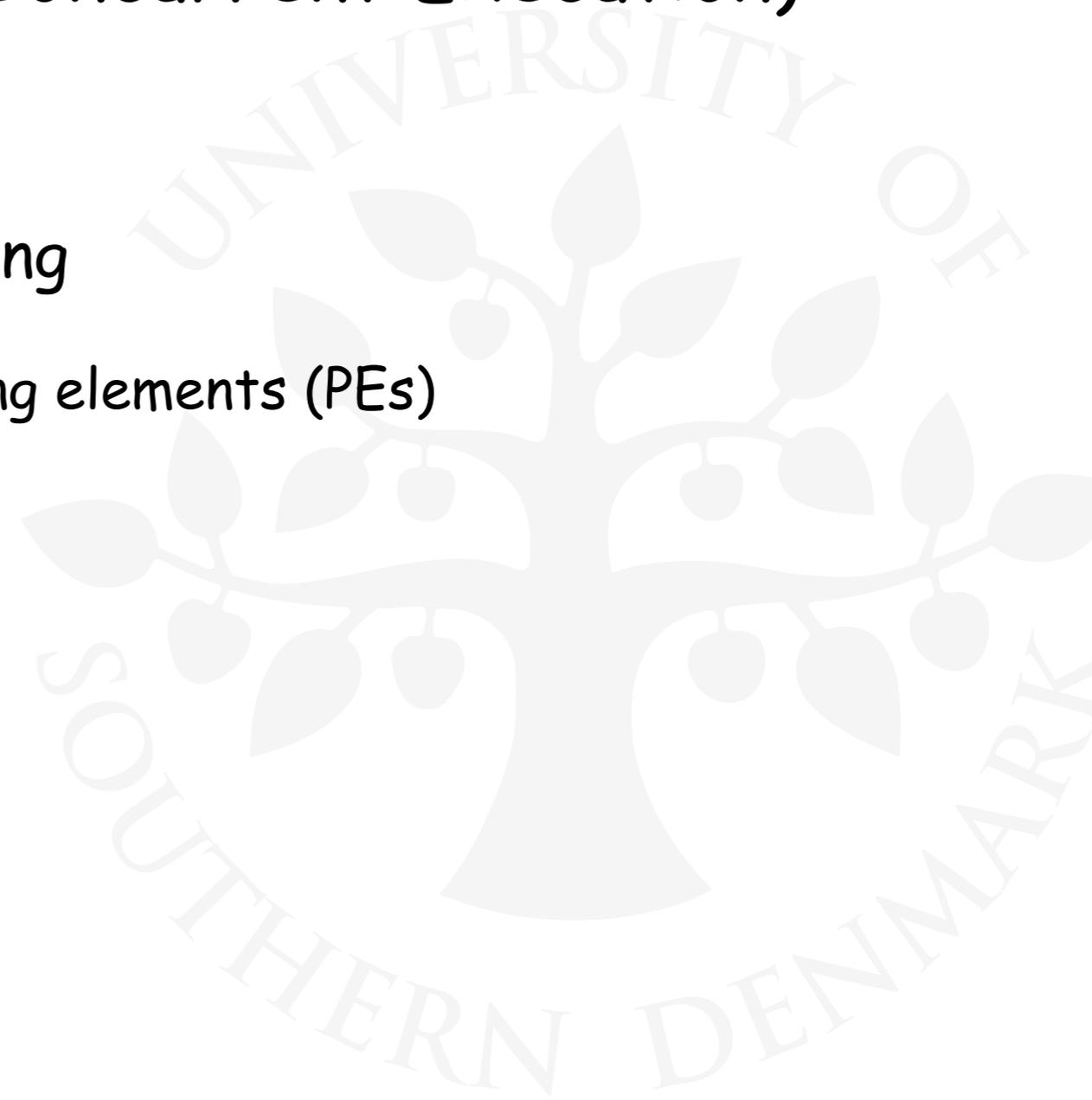


Definition: Concurrency

◆ Concurrency (aka. Pseudo-Concurrent Execution)

- Logically simultaneous processing

- ◆ Does not imply multiple processing elements (PEs)





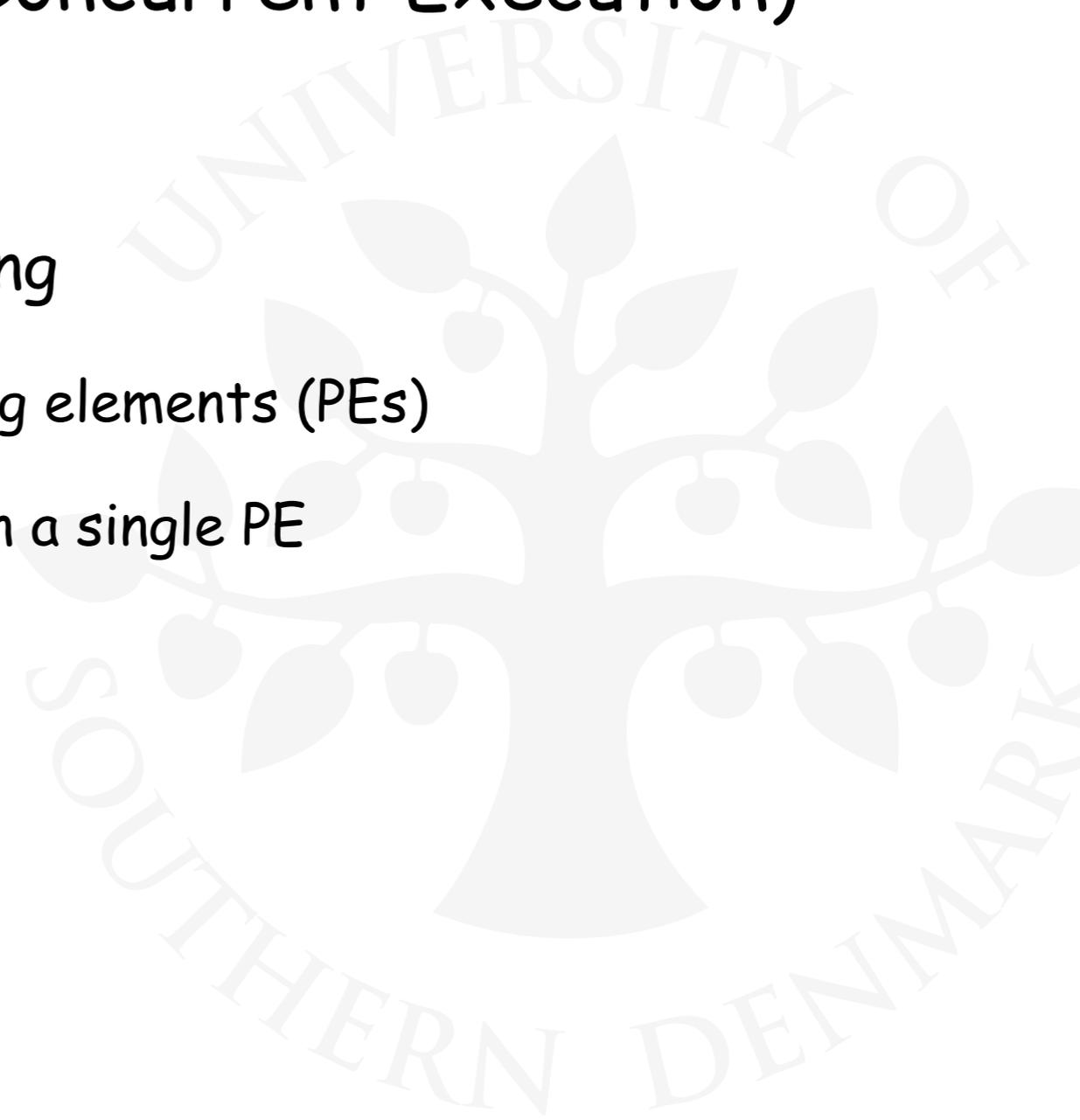
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- ◆ Does not imply multiple processing elements (PEs)

- ◆ Requires interleaved execution on a single PE



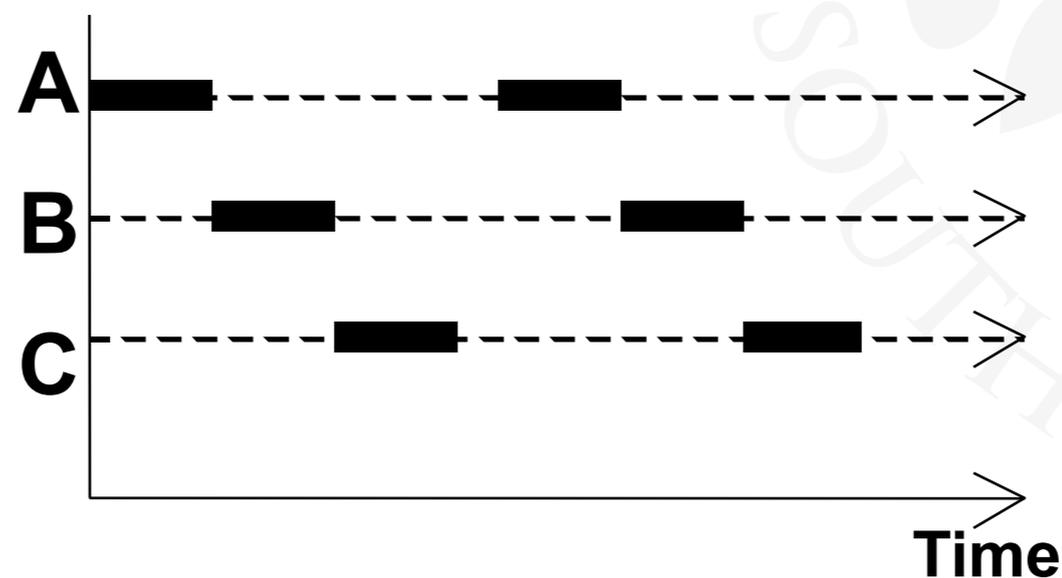
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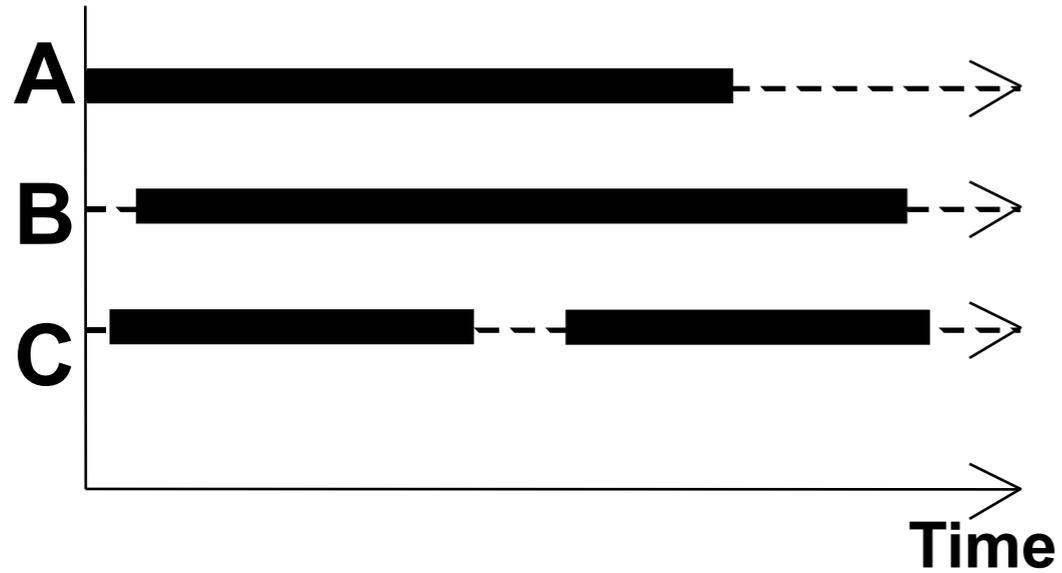
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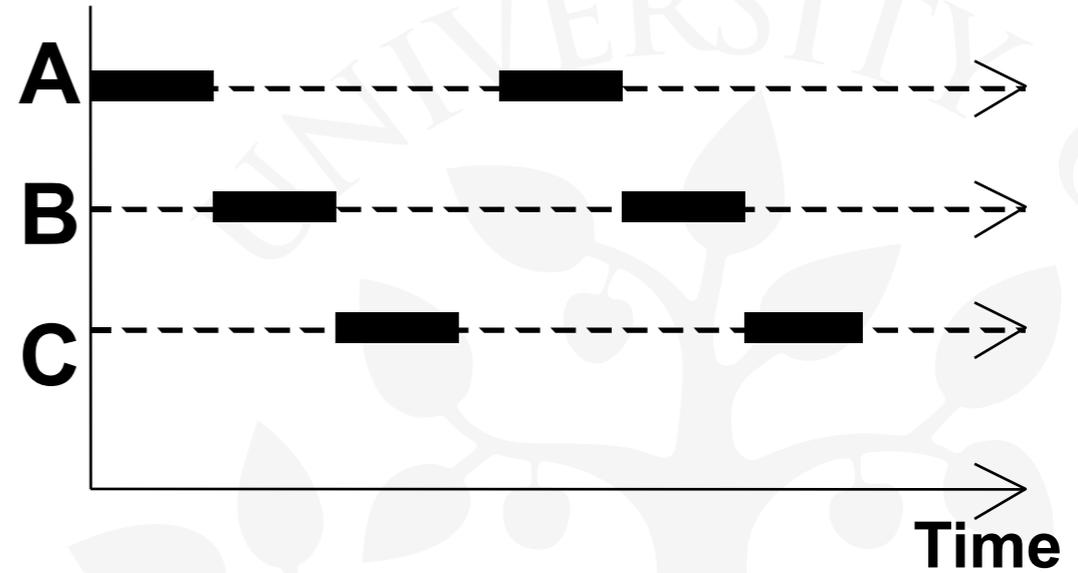


Parallelism vs Concurrency

◆ Parallelism



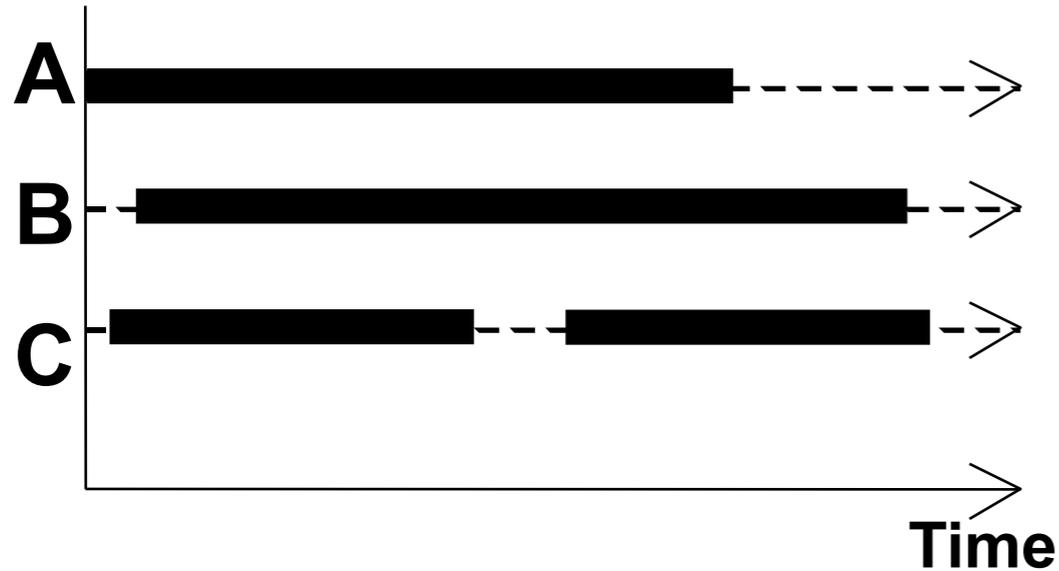
◆ Concurrency



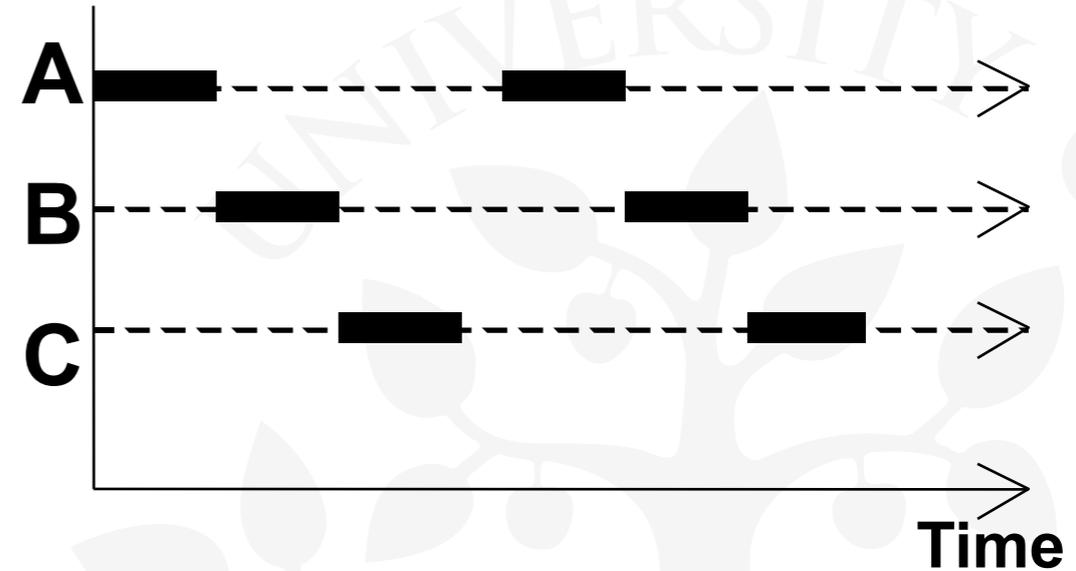


Parallelism vs Concurrency

◆ Parallelism



◆ Concurrency



Both **concurrency** and **parallelism** require controlled access to shared resources.

We use the terms parallel and concurrent interchangeably (and generally do not distinguish between real and pseudo-concurrent execution).

Also, creating software independent of the physical setup, makes us capable of deploying it on any platform.

3.1 Modelling Concurrency





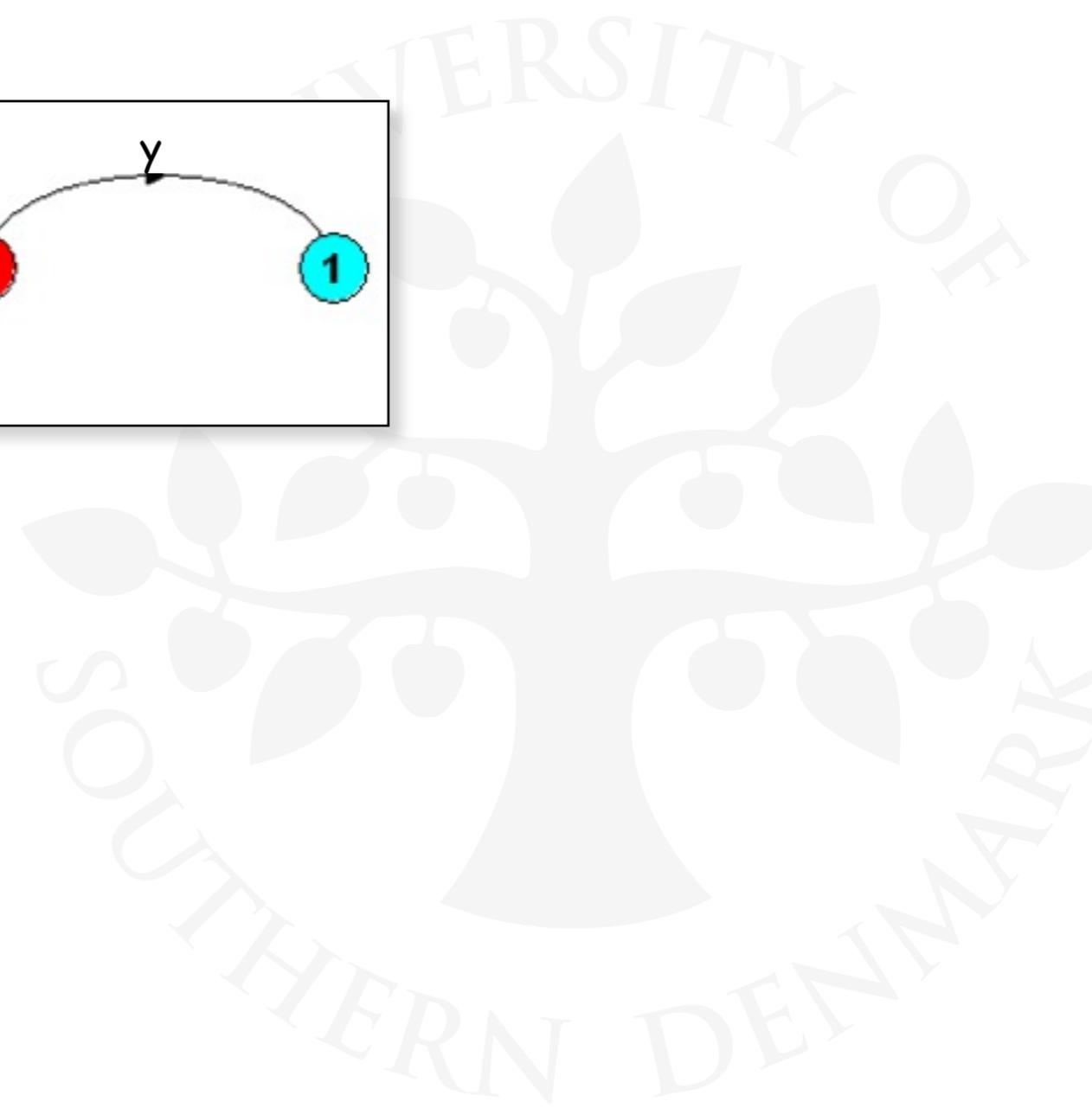
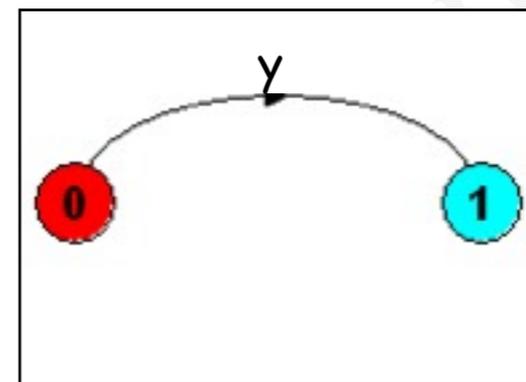
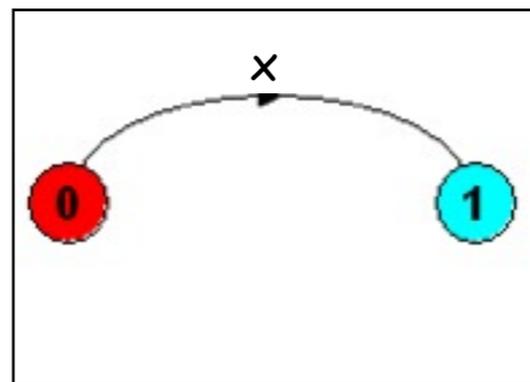
3.1 Modelling Concurrency

- ◆ How do we model concurrency?



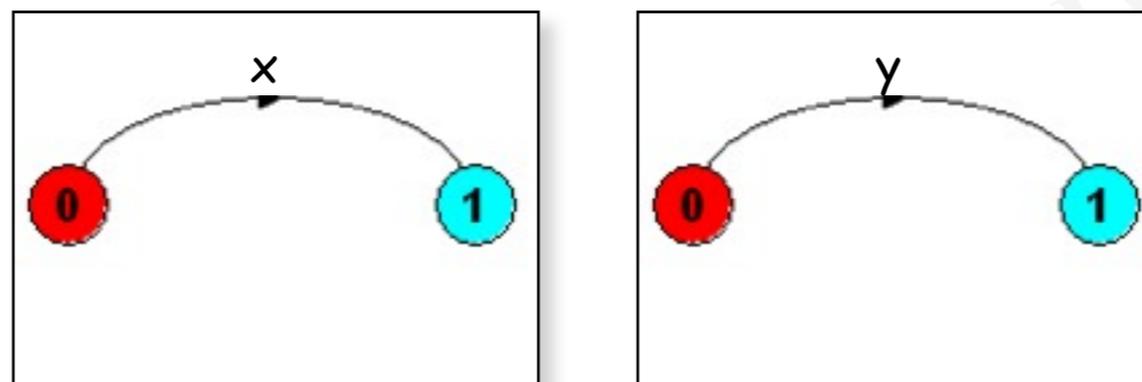
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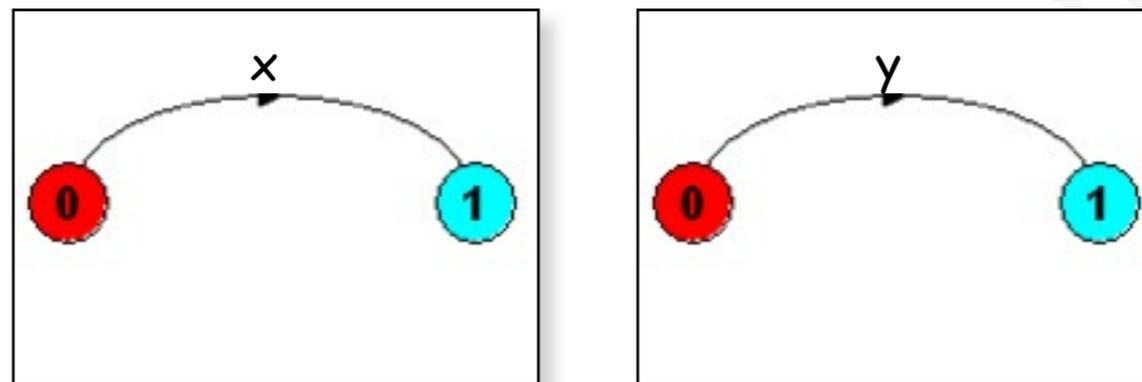
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Possible execution sequences?

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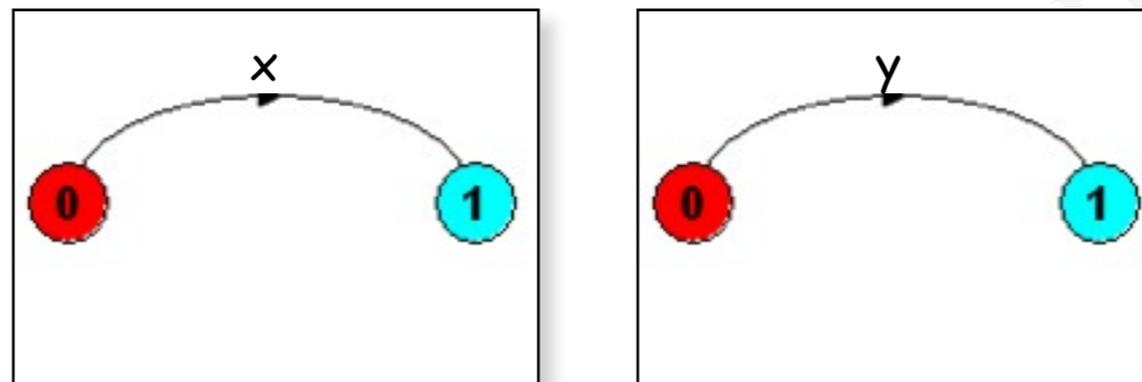


Possible execution sequences?

- $x ; y$

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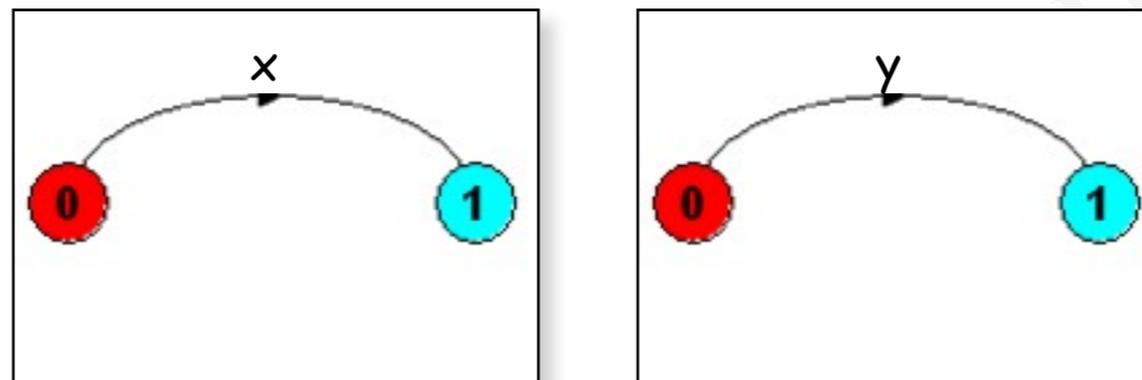


Possible execution sequences?

- $x ; y$
- $y ; x$

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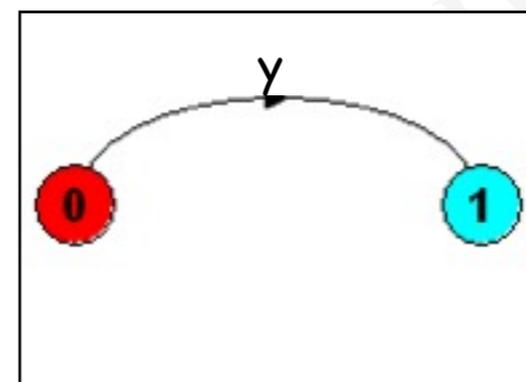
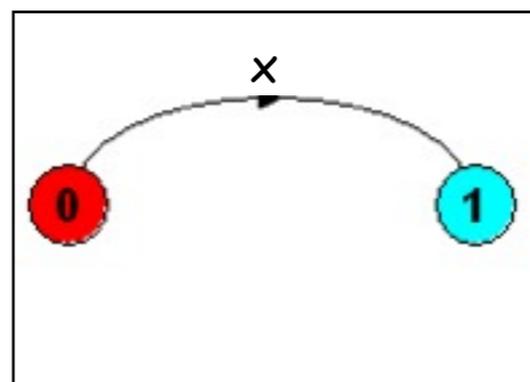


Possible execution sequences?

- $x ; y$
- $y ; x$
- $x \parallel y$

3.1 Modelling Concurrency

◆ How do we model concurrency?



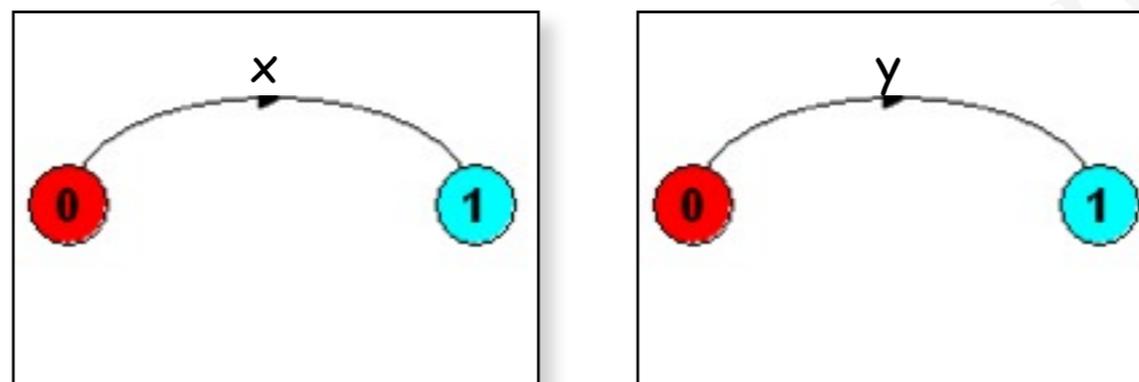
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Asynchronous
model of execution

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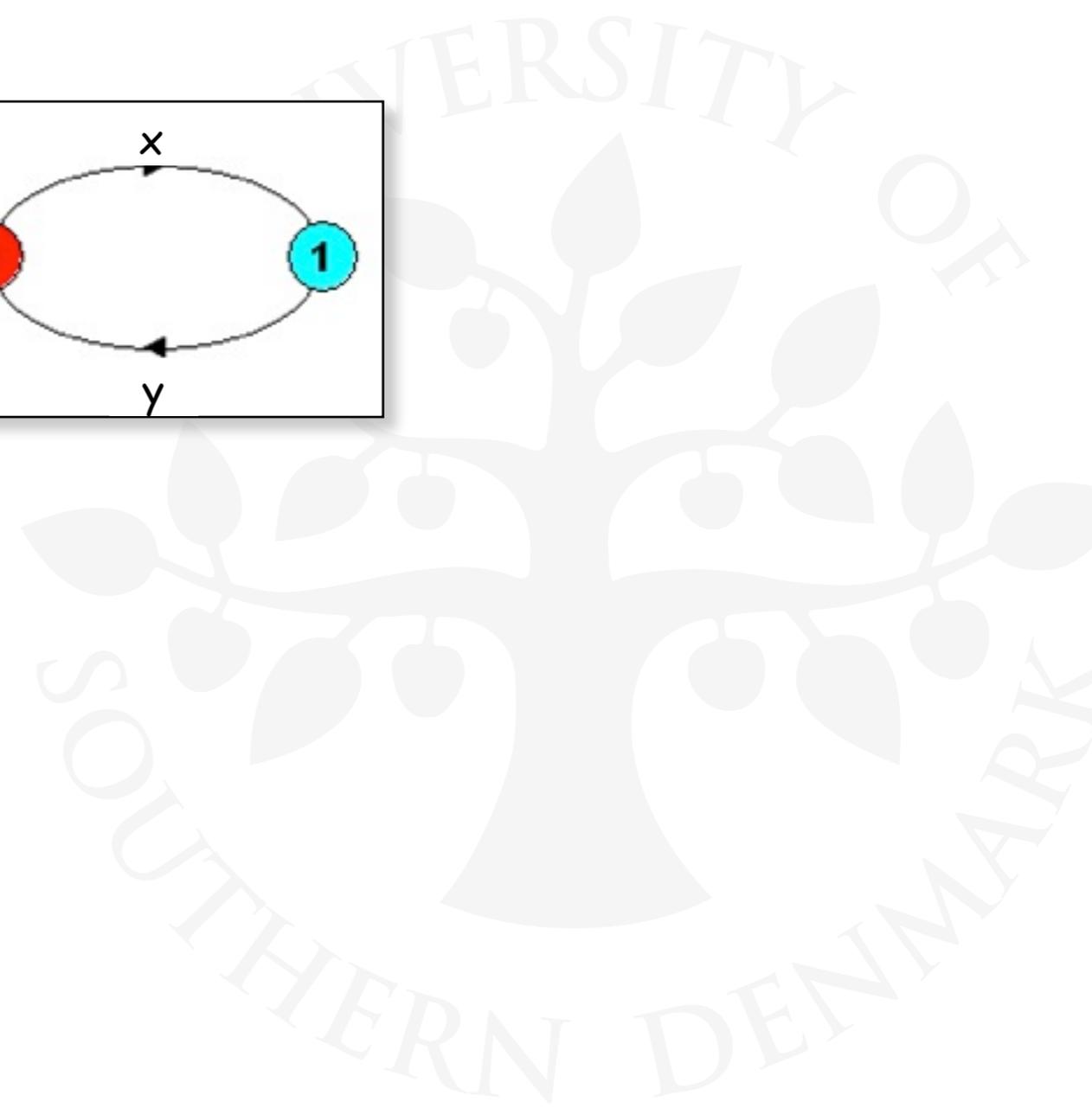
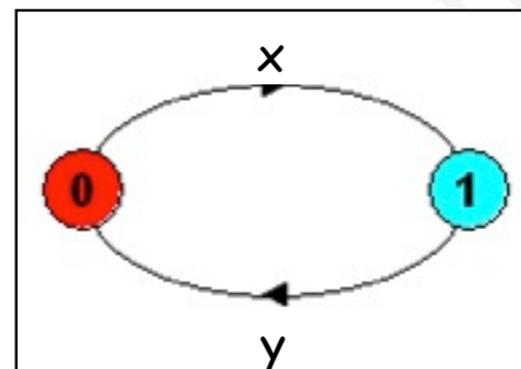
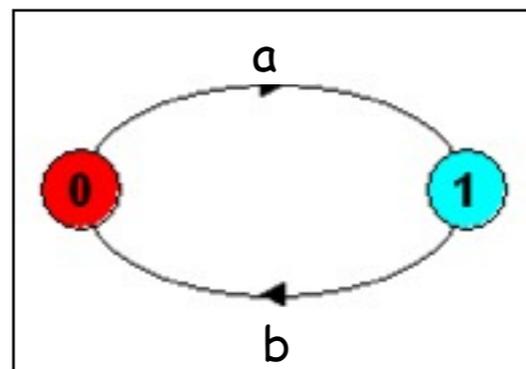
- Arbitrary relative order of actions from different processes (**interleaving** but preservation of each process order)

3.1 Modelling Concurrency



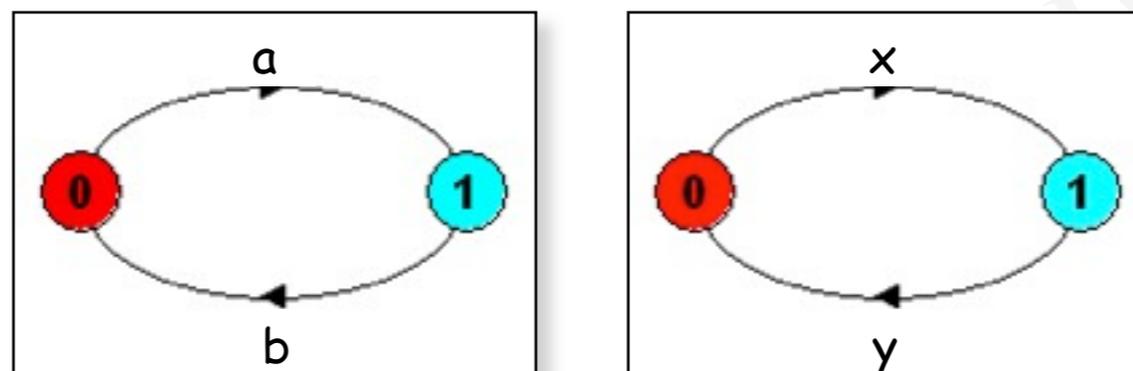
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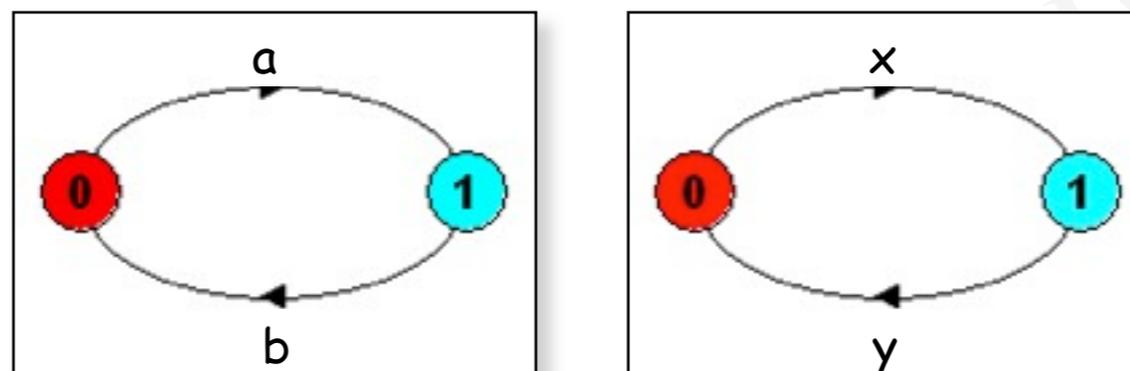
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- We choose to abstract away time:

3.1 Modelling Concurrency

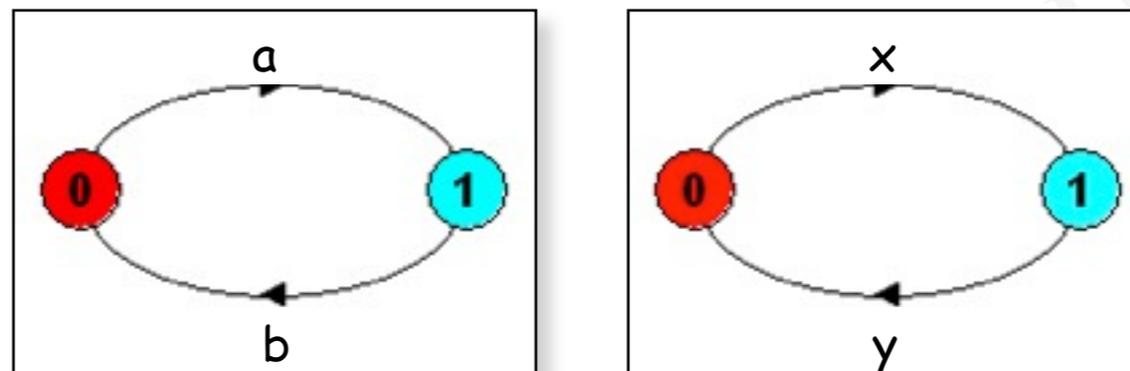
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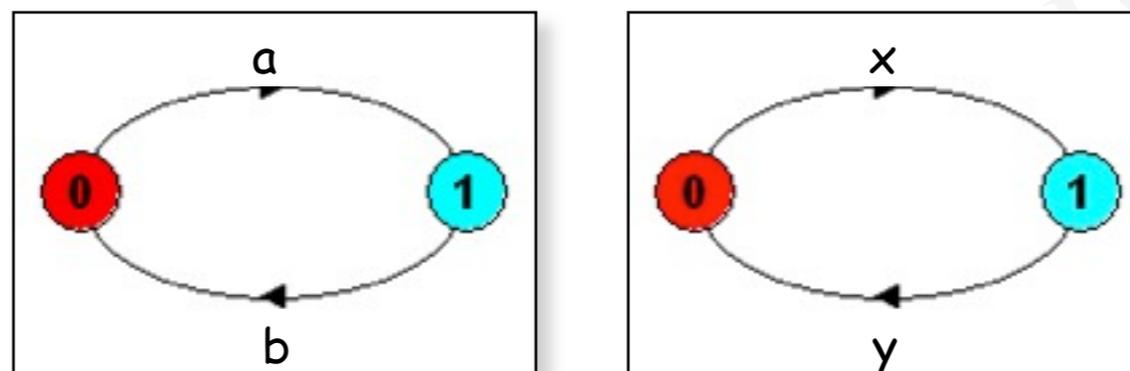
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3.1 Modelling Concurrency

- ◆ How should we model process execution speed?



- We choose to abstract away time:

- ◆ Arbitrary speed!

-: we can say nothing of real-time properties

+: independent of architecture, processor speed, scheduling policies, ...

Parallel Composition - Action Interleaving



If P and Q are processes then $(P||Q)$ represents the concurrent execution of P and Q . The operator ' $||$ ' is the parallel composition operator.





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Possible traces as
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- `scratch → think → talk`



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- `scratch` → `think` → `talk`
- `think` → `scratch` → `talk`



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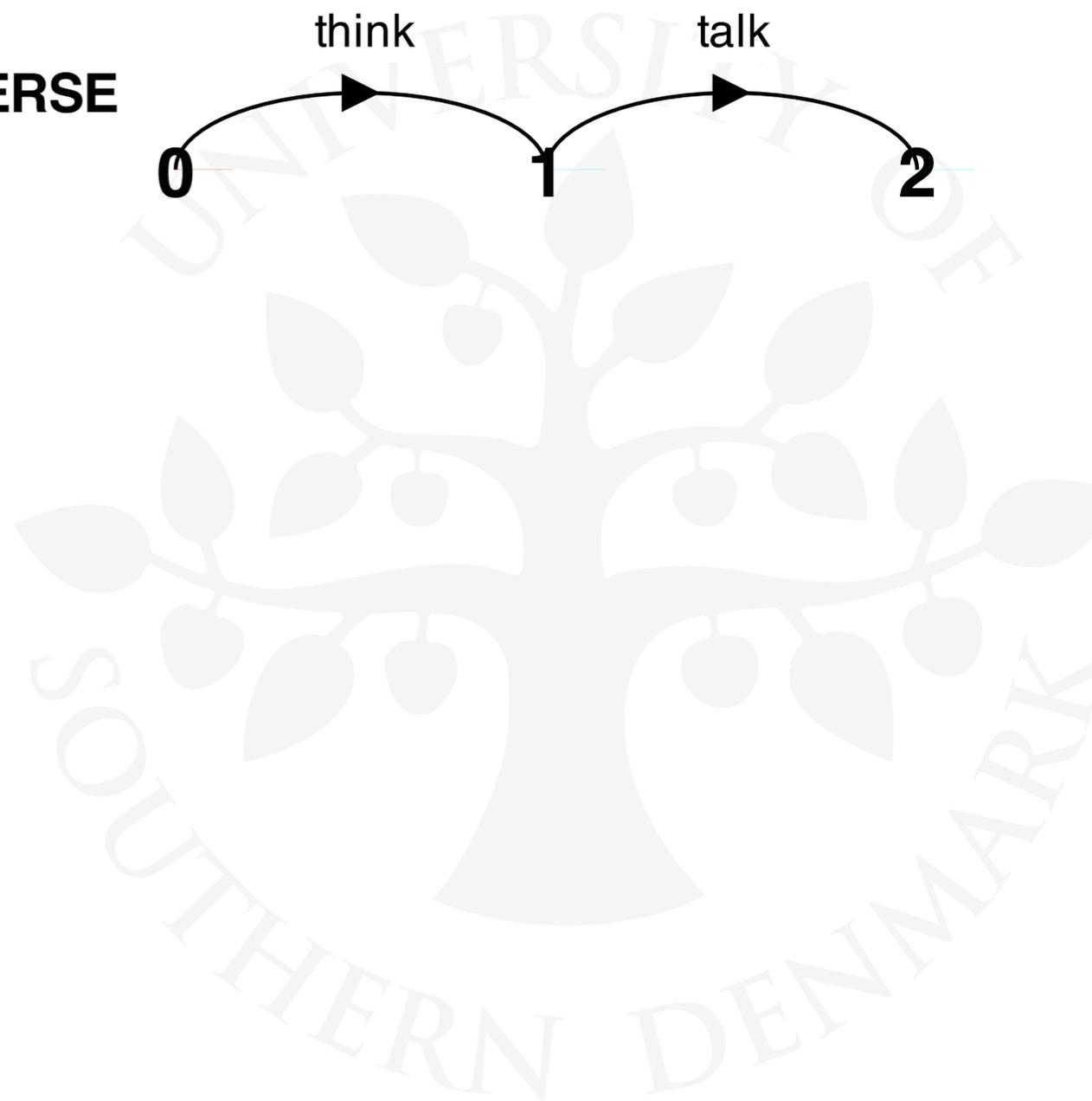
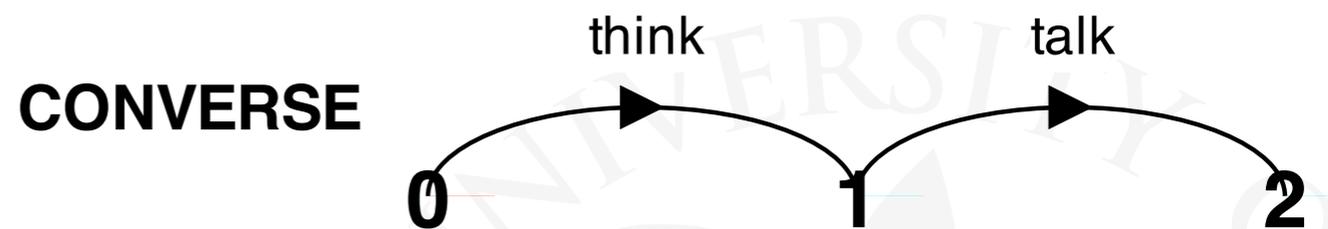
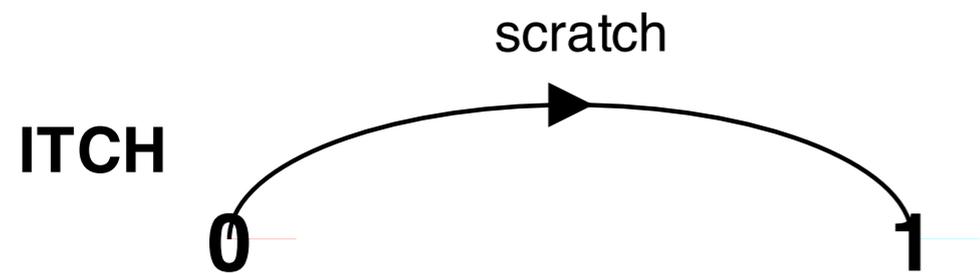
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- `think` → `scratch` → `talk`
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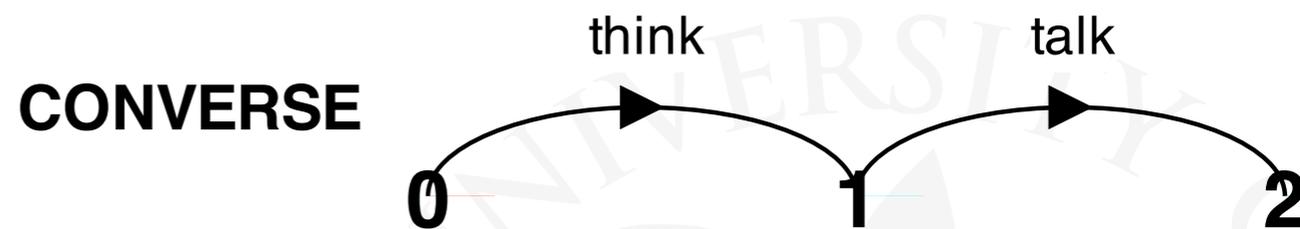
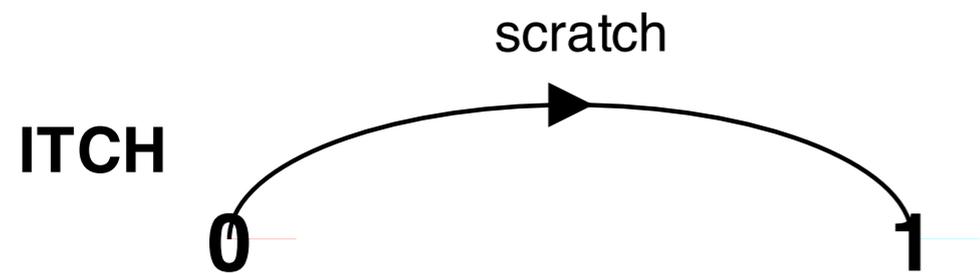


Parallel Composition - Action Interleaving

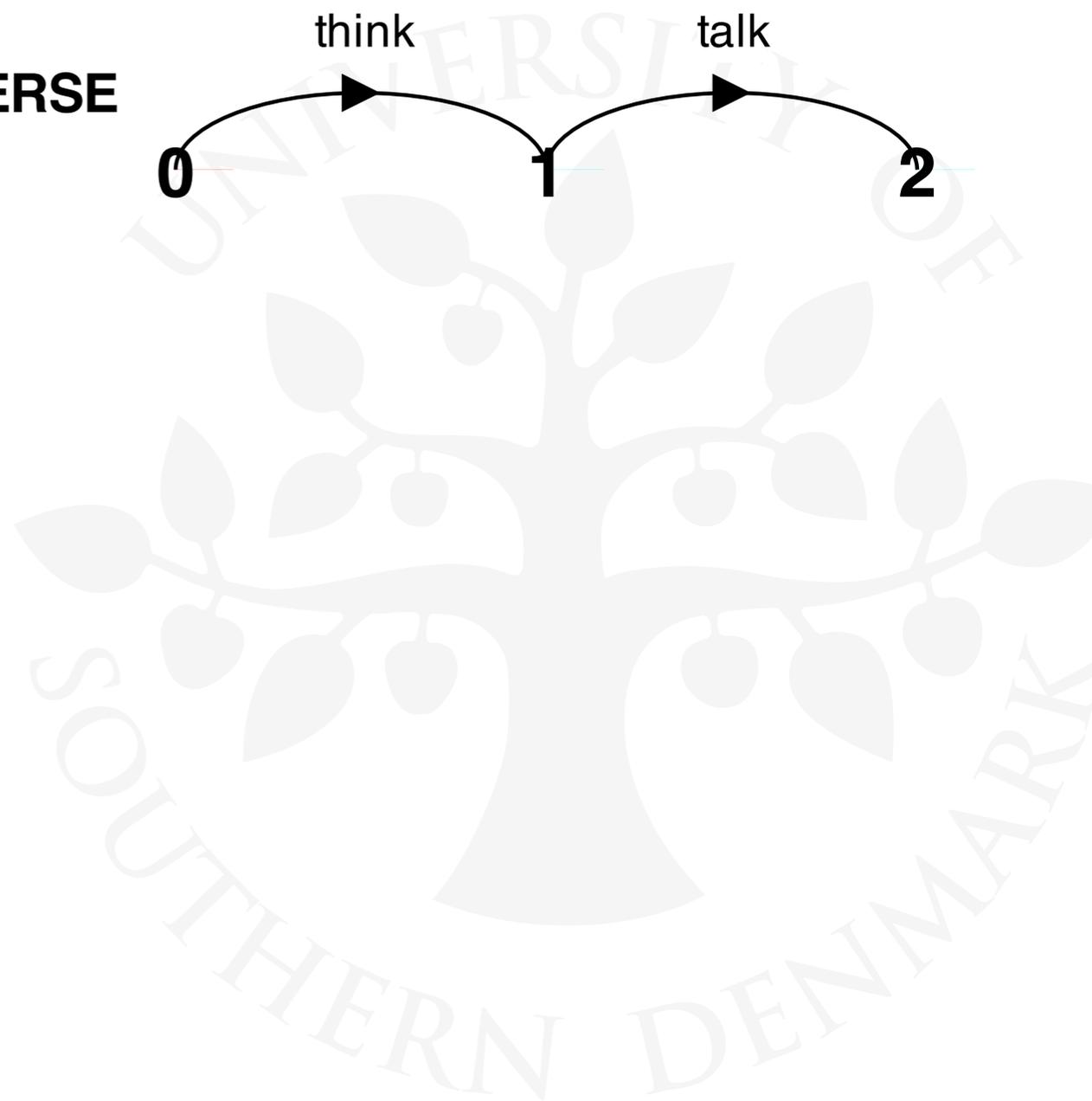




Parallel Composition - Action Interleaving

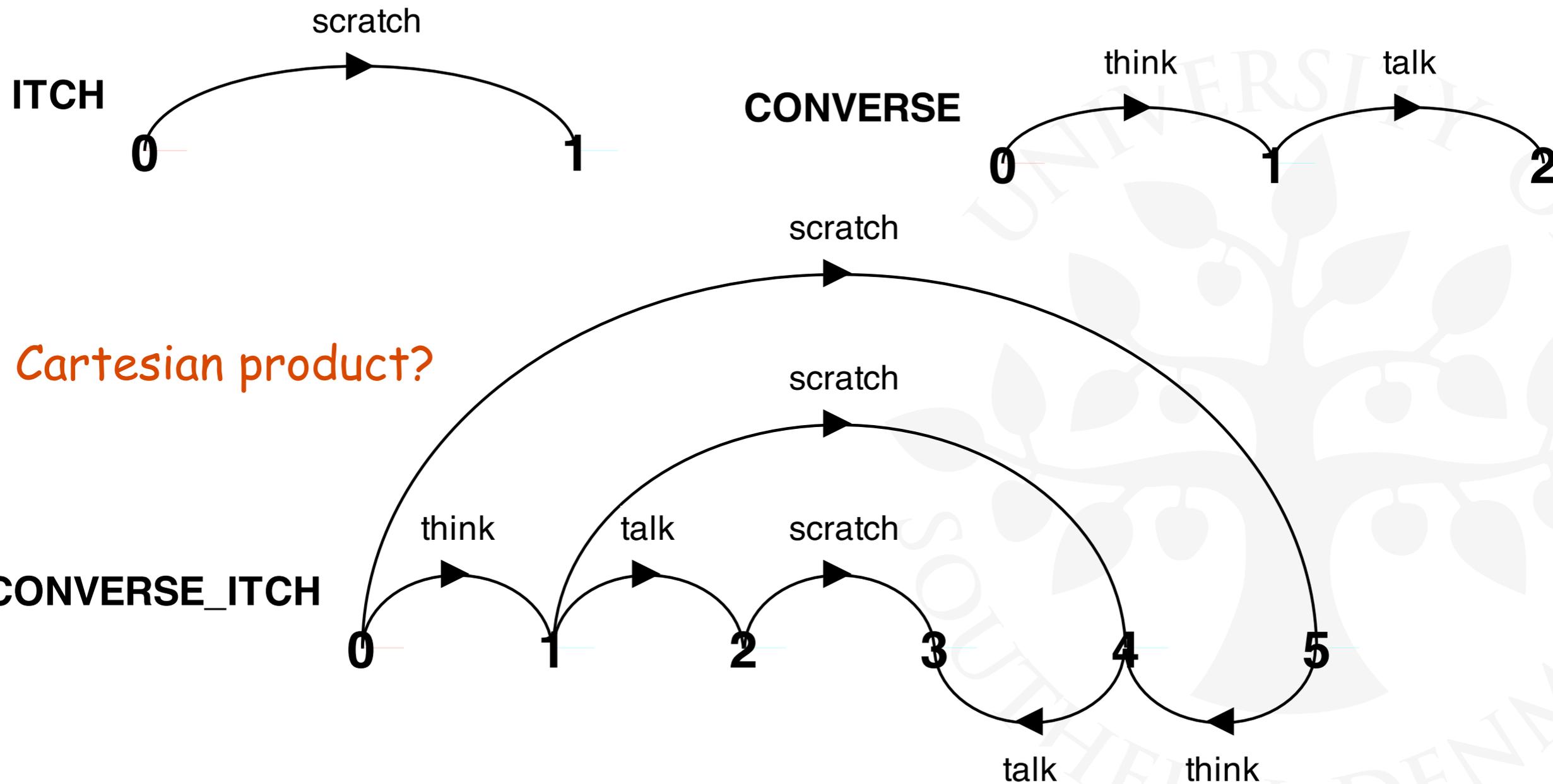


Cartesian product?



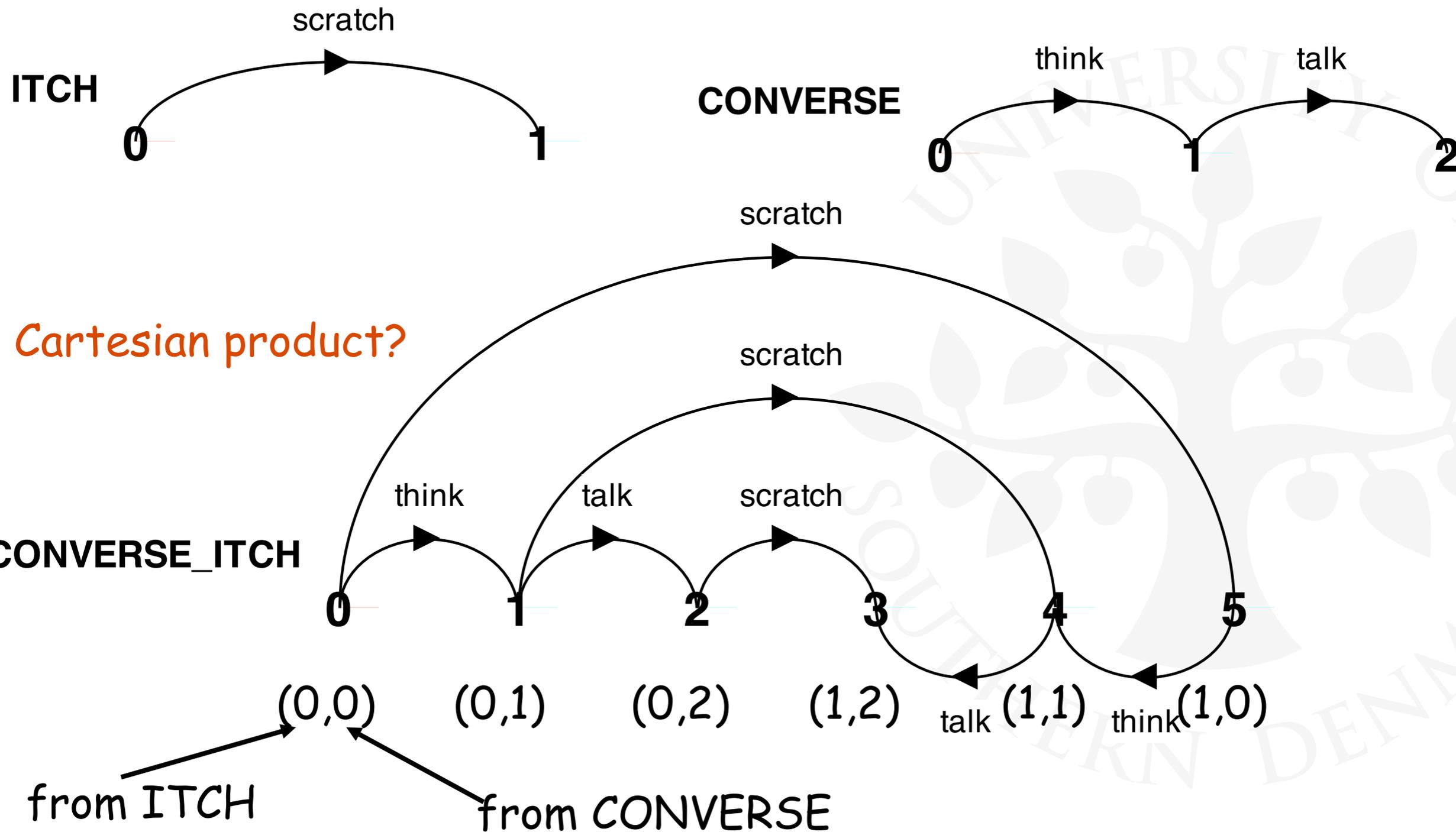


Parallel Composition - Action Interleaving





Parallel Composition - Action Interleaving

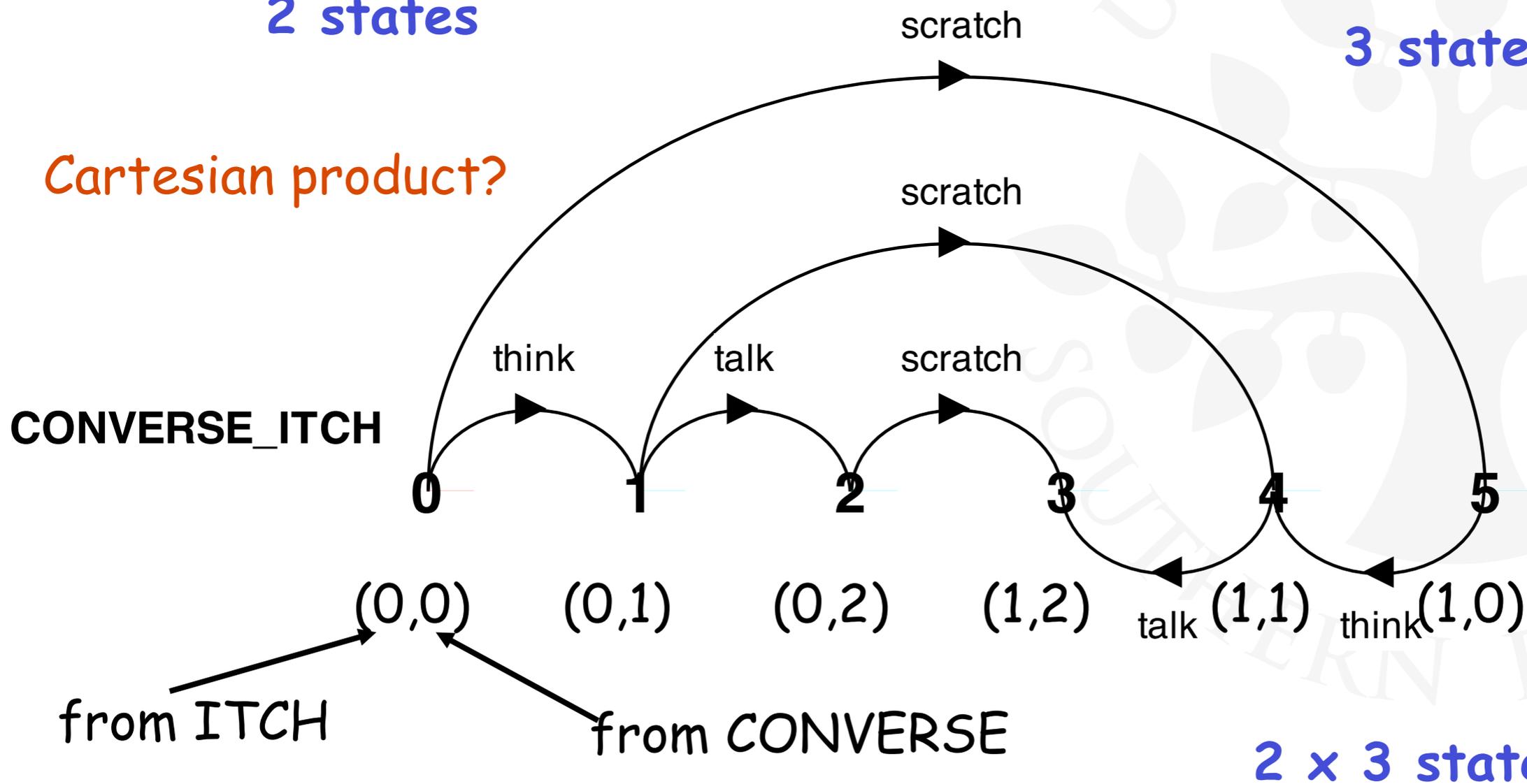




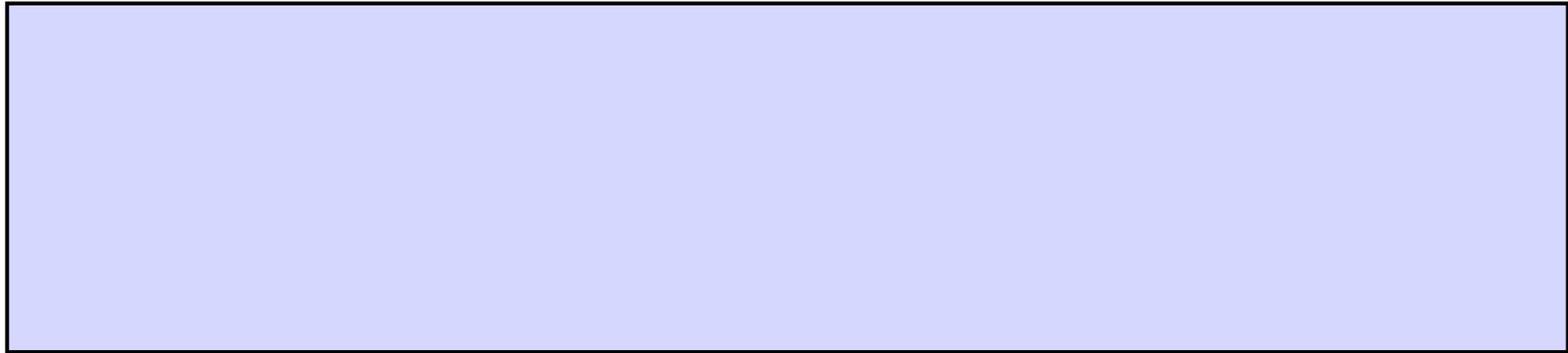
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Cartesian product?



Parallel Composition - Algebraic Laws



Parallel Composition - Algebraic Laws



Commutative: $(P \parallel Q) = (Q \parallel P)$

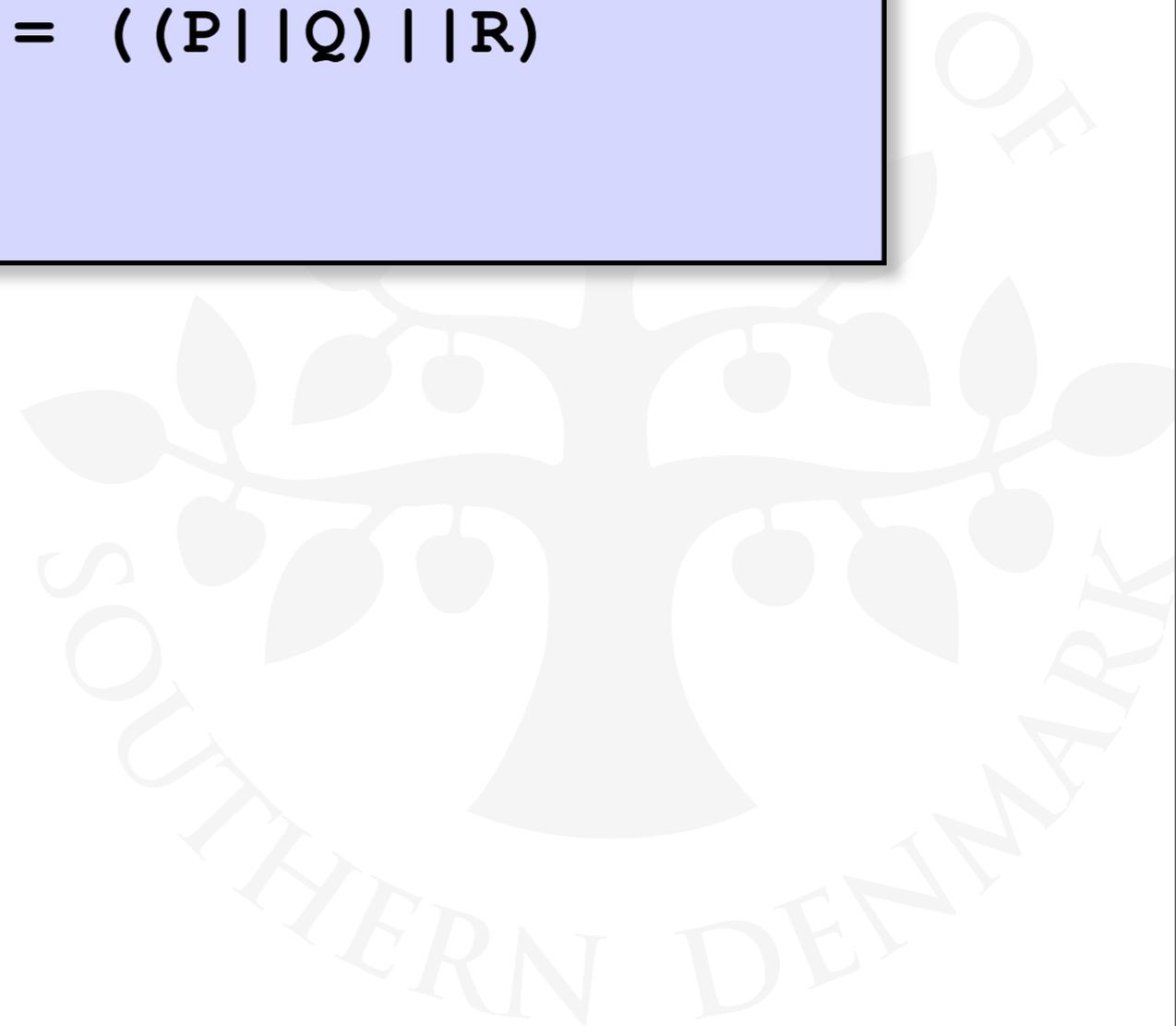




Parallel Composition - Algebraic Laws

Commutative: $(P \parallel Q) = (Q \parallel P)$

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Small example:





Parallel Composition - Algebraic Laws

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Small example:

```
MALTHE = (climbTree->fall->MALTHE) .
```

```
OSKAR = (run->jump->OSKAR) .
```

```
||MALTHE_OSKAR = (MALTHE || OSKAR) .
```



Parallel Composition - Algebraic Laws

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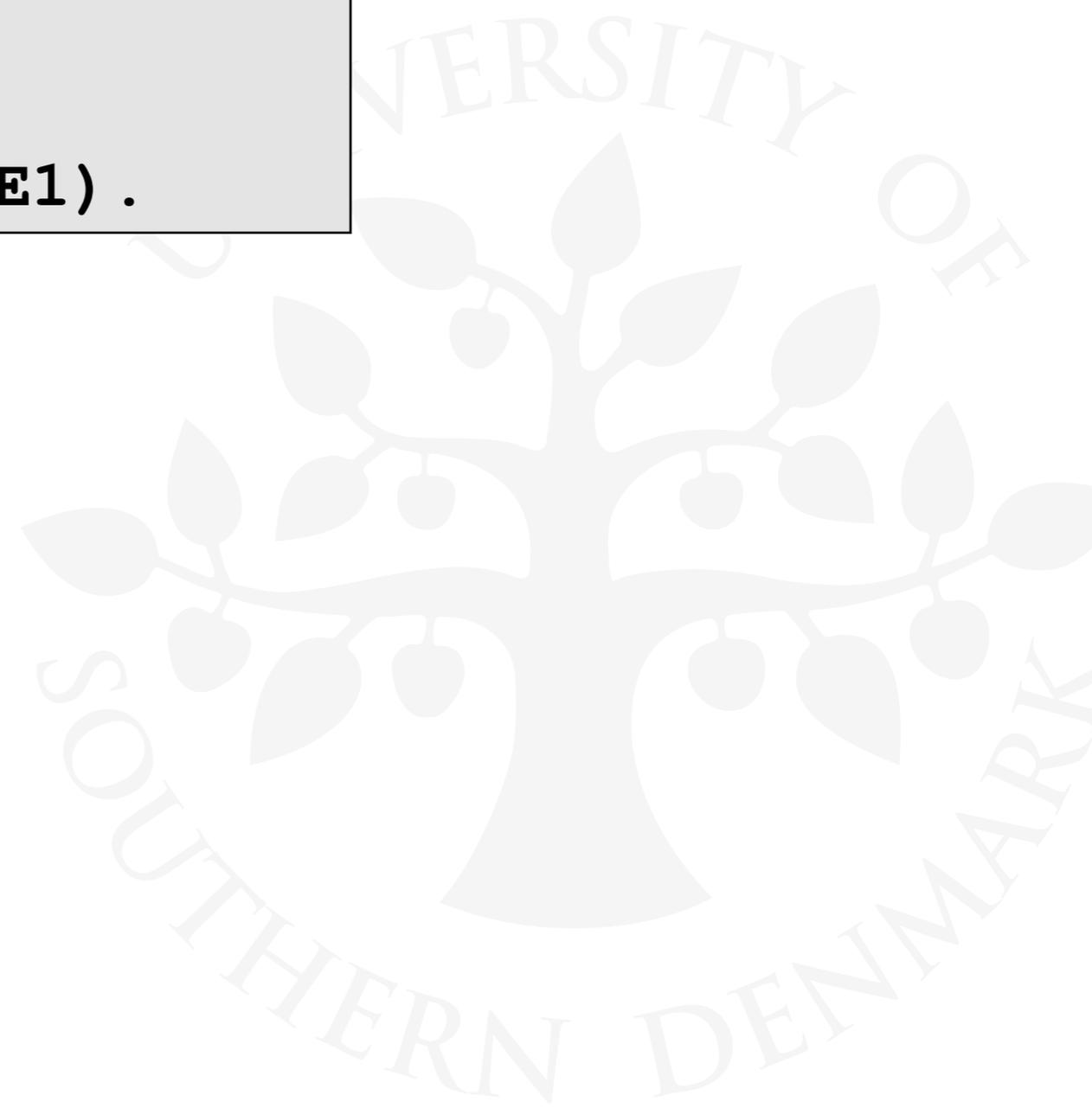
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LTS? Traces? Number of states?



Modelling Interaction - Shared Actions

```
MAKE1 = (make->ready->STOP) .  
USE1  = (ready->use->STOP) .  
  
||MAKE1_USE1 = (MAKE1 || USE1) .
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Modelling Interaction - Shared Actions

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◆ Shared Actions:

If processes in a composition have actions in common, these actions are said to be **shared**.

Shared actions are the way that process interaction is modelled. While unshared actions may be arbitrarily interleaved, a shared action **must be executed at the same time by all processes** that participate in the shared action.



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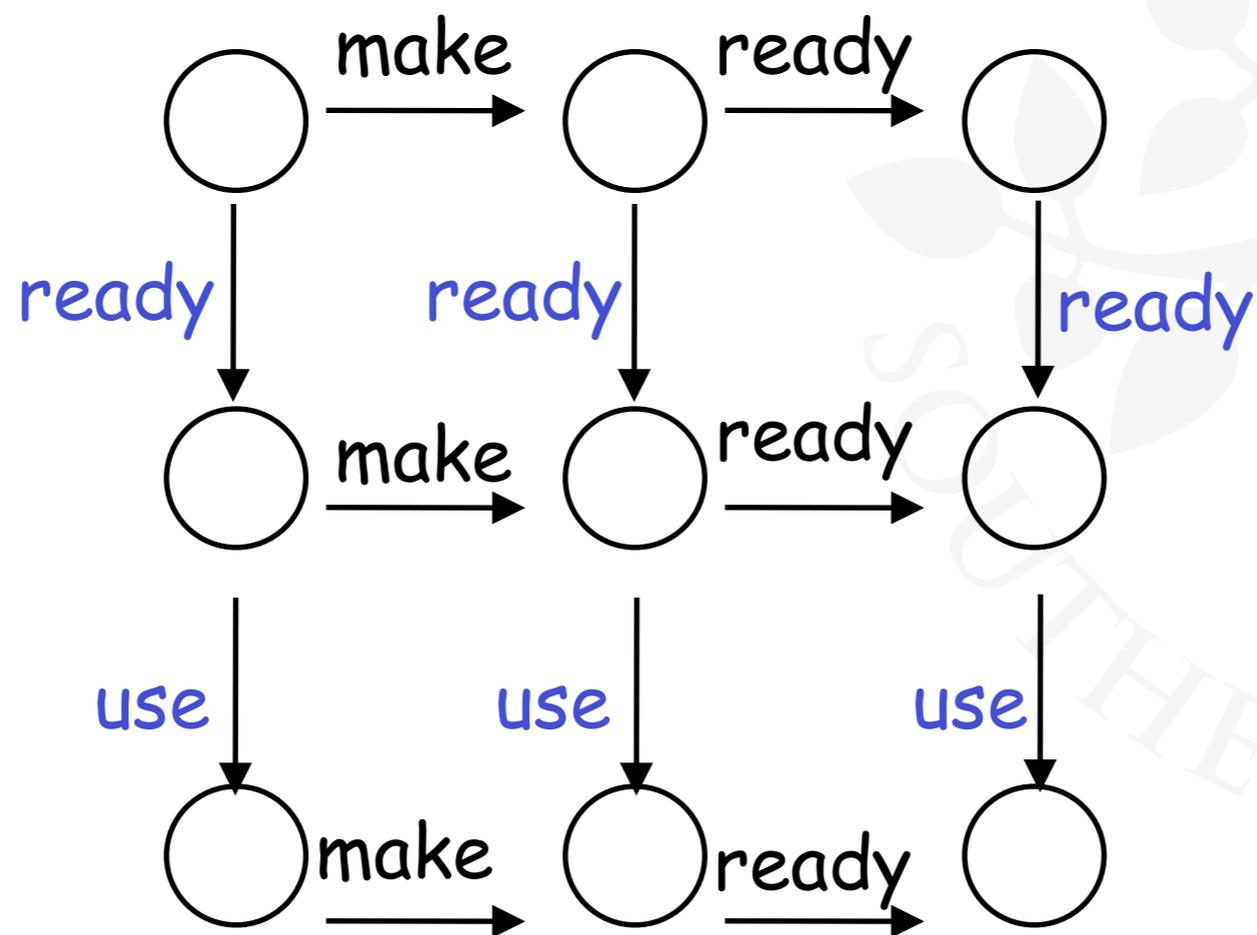
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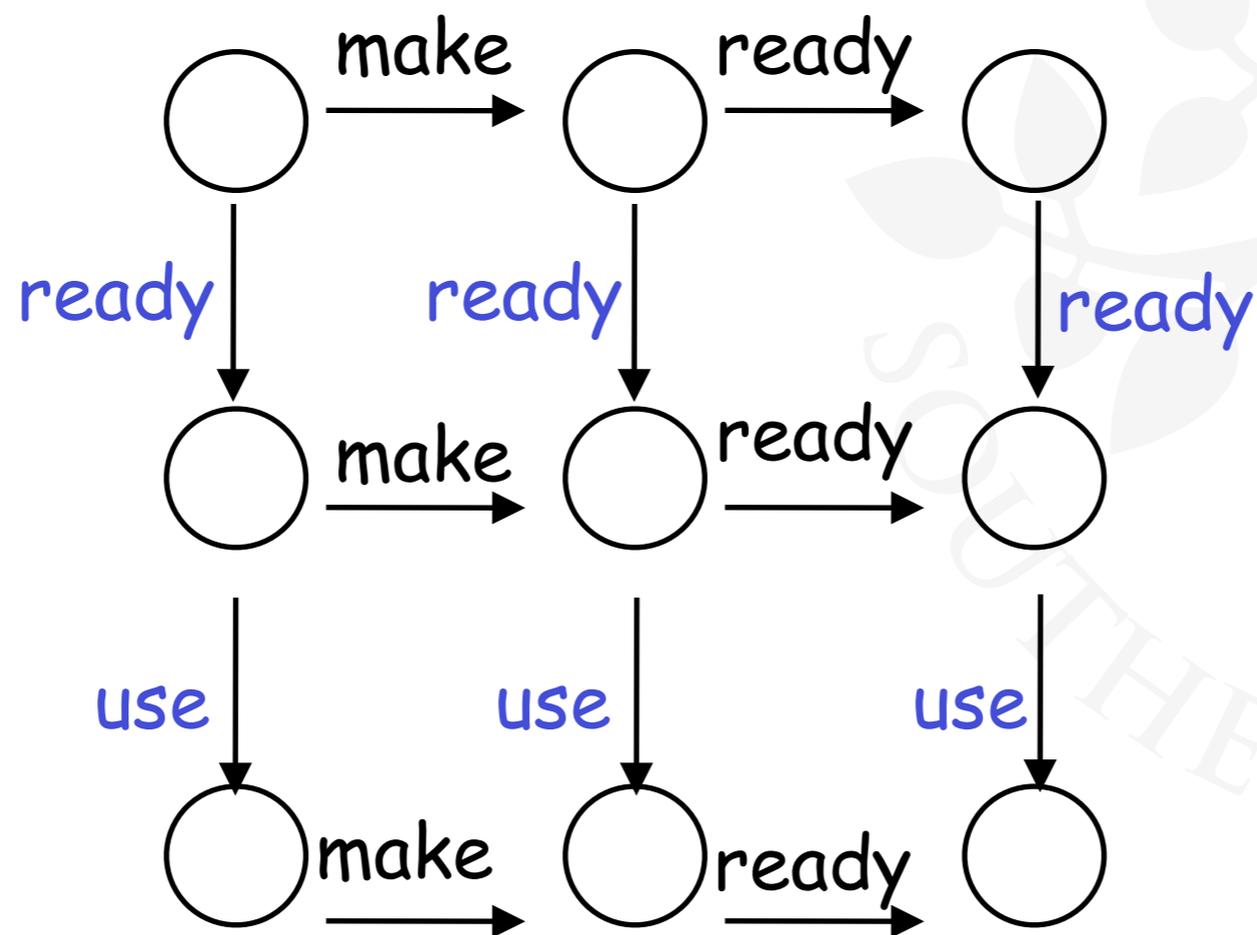
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3 states

3 states





Modelling Interaction - Example

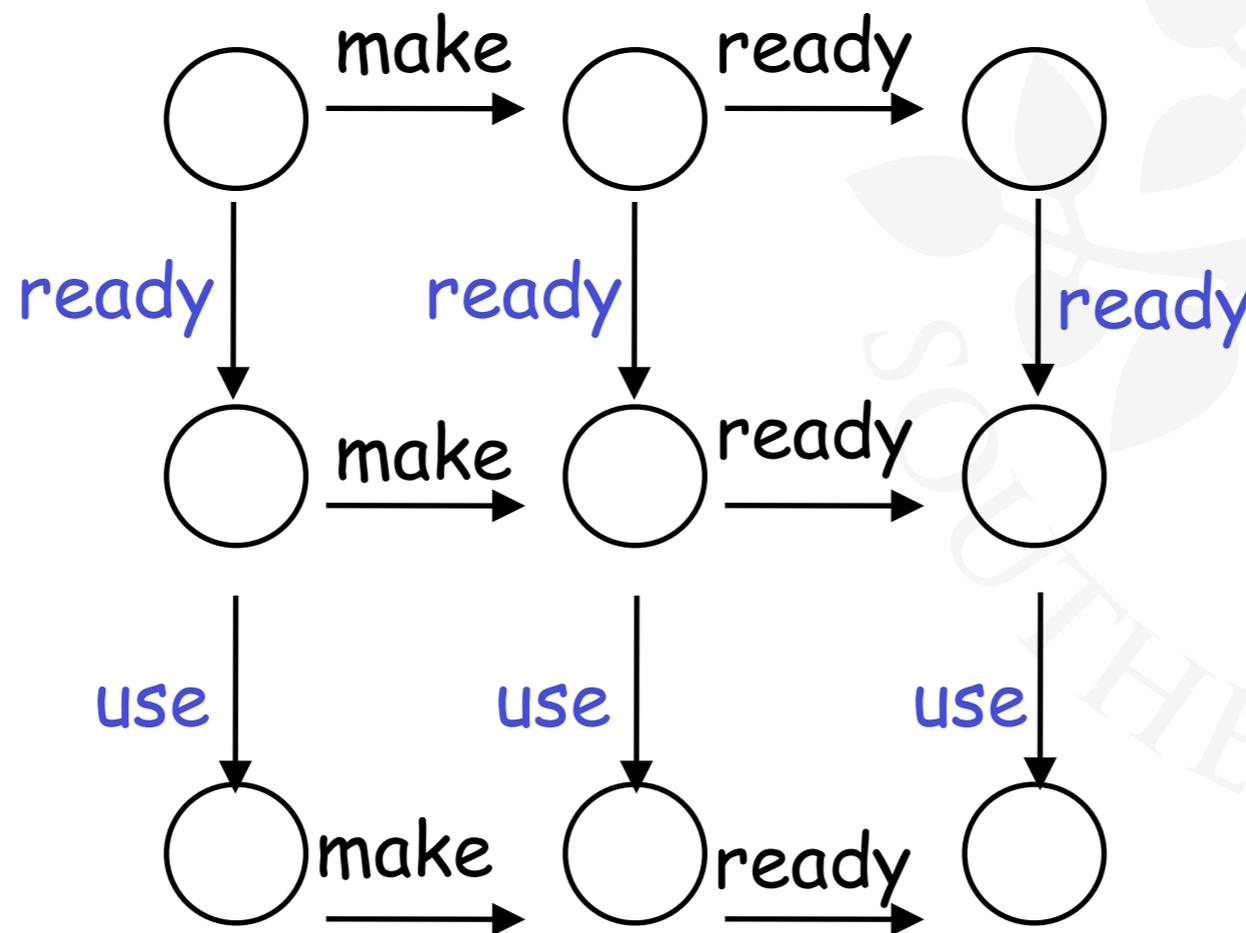
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3 x 3 states?



Modelling Interaction - Example

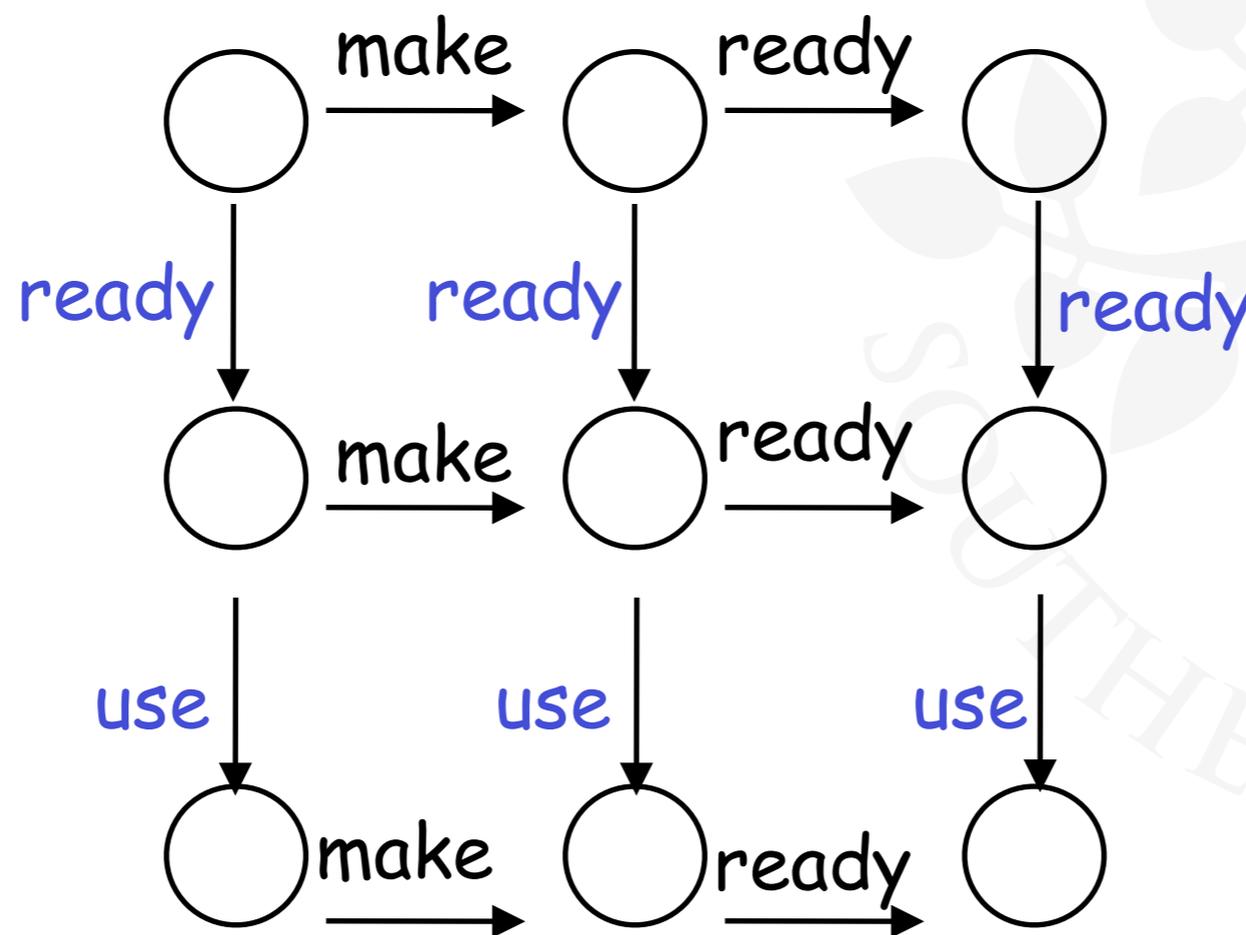
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3 states

3 states



3 x 3 states?

No...!

Modelling Interaction - Example

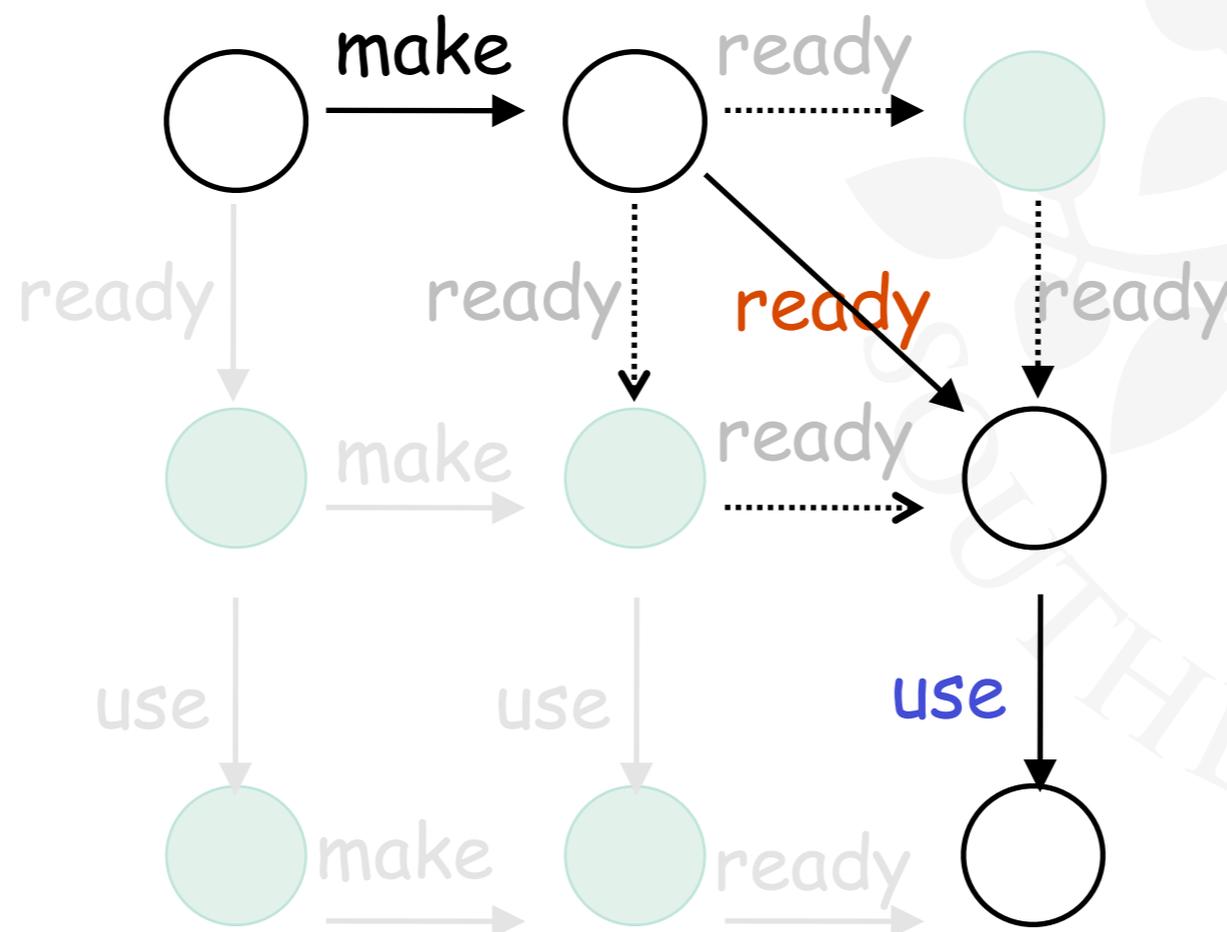
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3 states

3 states





Modelling Interaction - Example

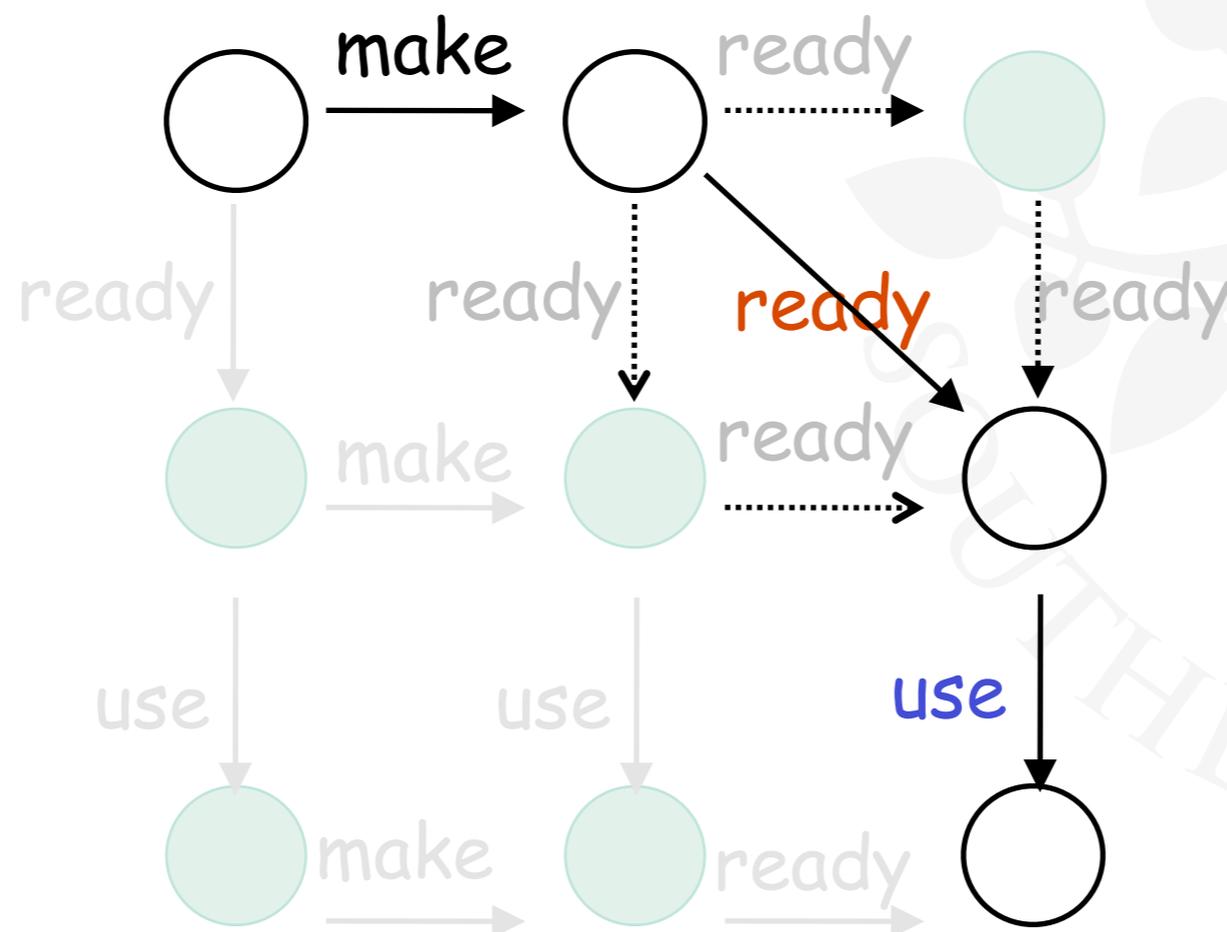
MAKE1 = (make->*ready*->STOP) .

USE1 = (*ready*->use->STOP) .

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3 states

3 states



4 states!

Interaction may constrain the overall behaviour !



Example

$$P = (x \rightarrow y \rightarrow P) .$$
$$Q = (y \rightarrow x \rightarrow Q) .$$
$$||R = (P || Q) .$$

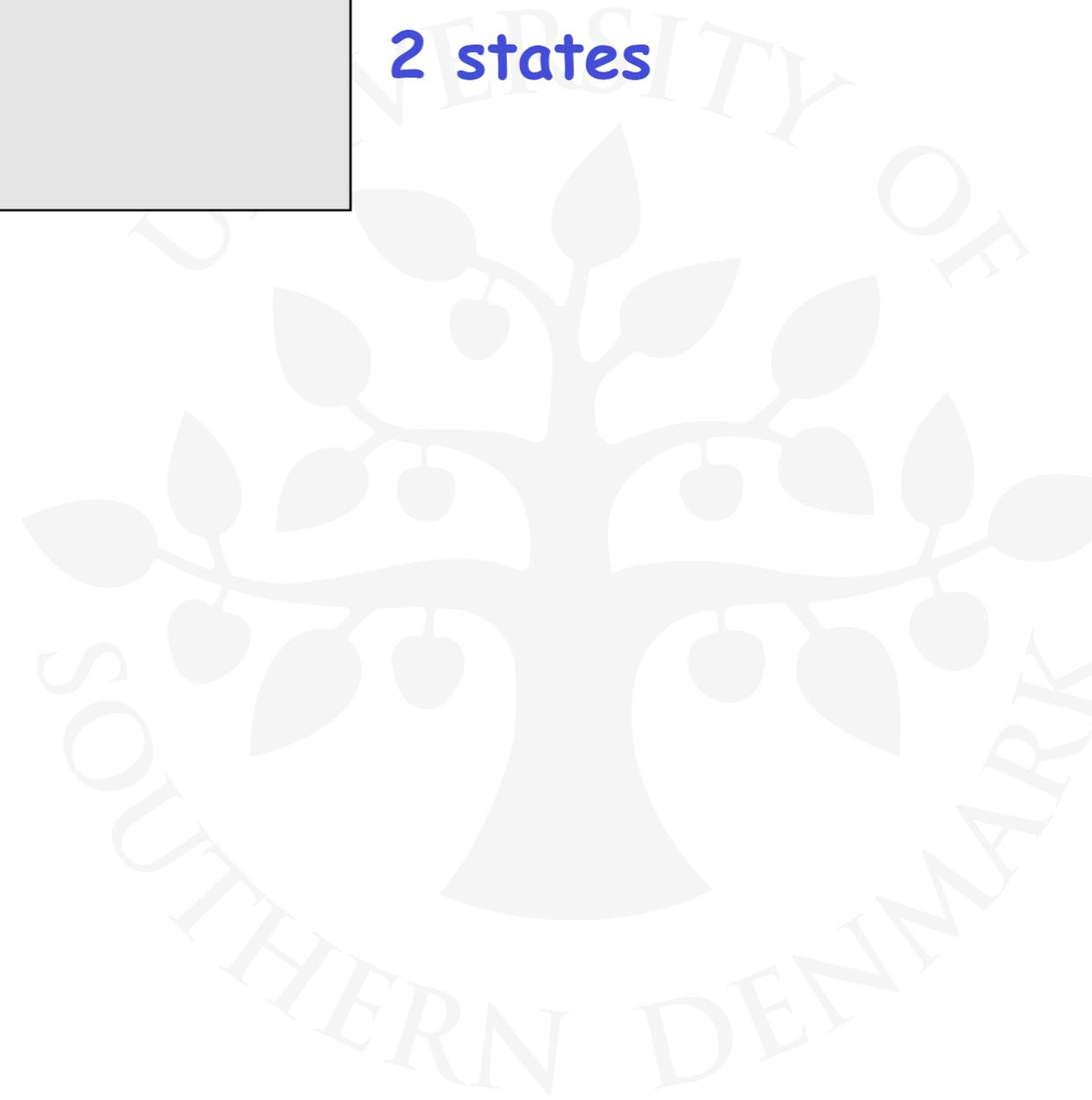



Example

$$P = (x \rightarrow y \rightarrow P) .$$
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2 states

2 states





Example

$$P = (x \rightarrow y \rightarrow P) .$$
$$Q = (y \rightarrow x \rightarrow Q) .$$
$$P \parallel Q = (P \parallel Q) .$$

2 states

2 states

LTS? Traces? Number of states?



Example

$$P = (x \rightarrow y \rightarrow P) .$$
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2 states

2 states

LTS? Traces? Number of states?

$$P = (a \rightarrow P \mid b \rightarrow P) .$$
$$Q = (c \rightarrow Q) + \{a\} .$$
$$||PQ = (P || Q) .$$



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2 states

2 states

LTS? Traces? Number of states?

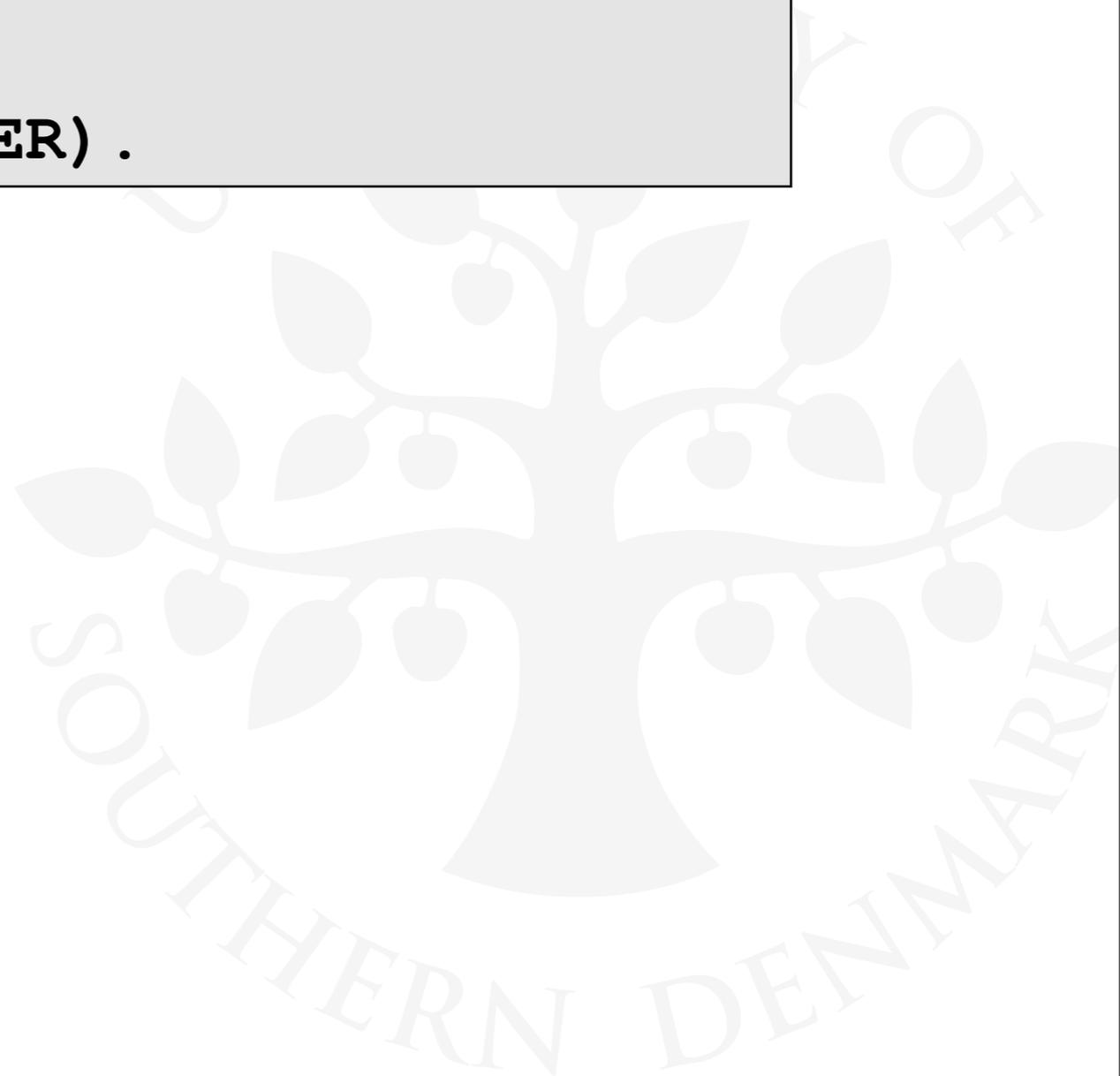
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Modelling Interaction - Example

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Modelling Interaction - Example

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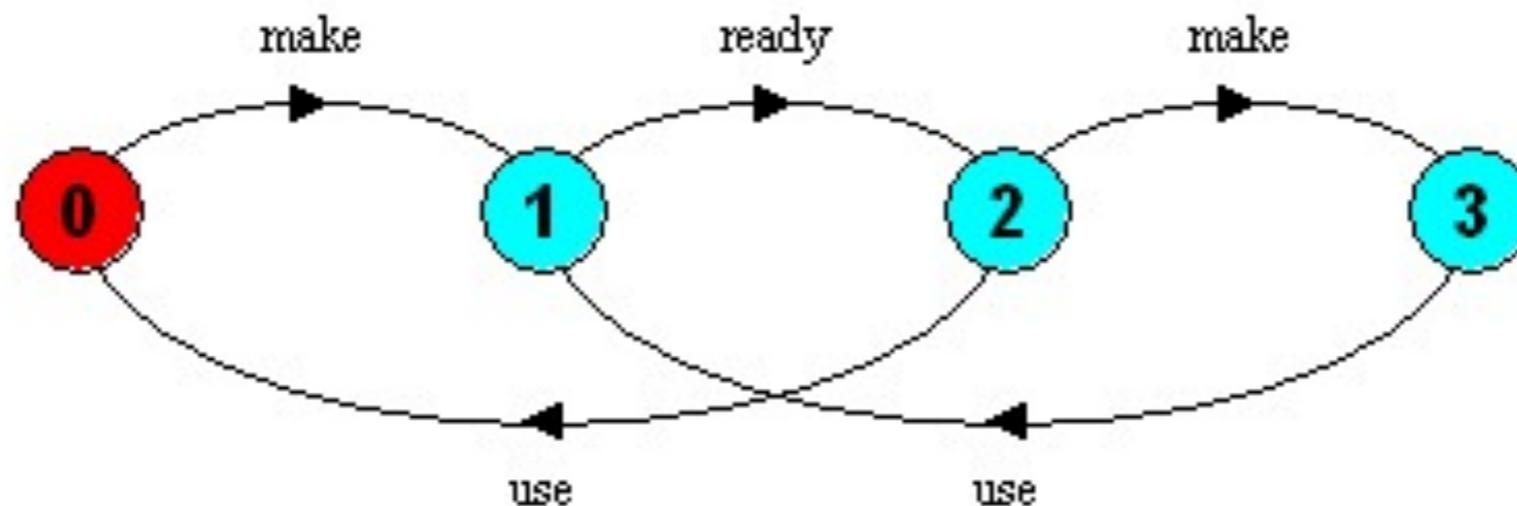
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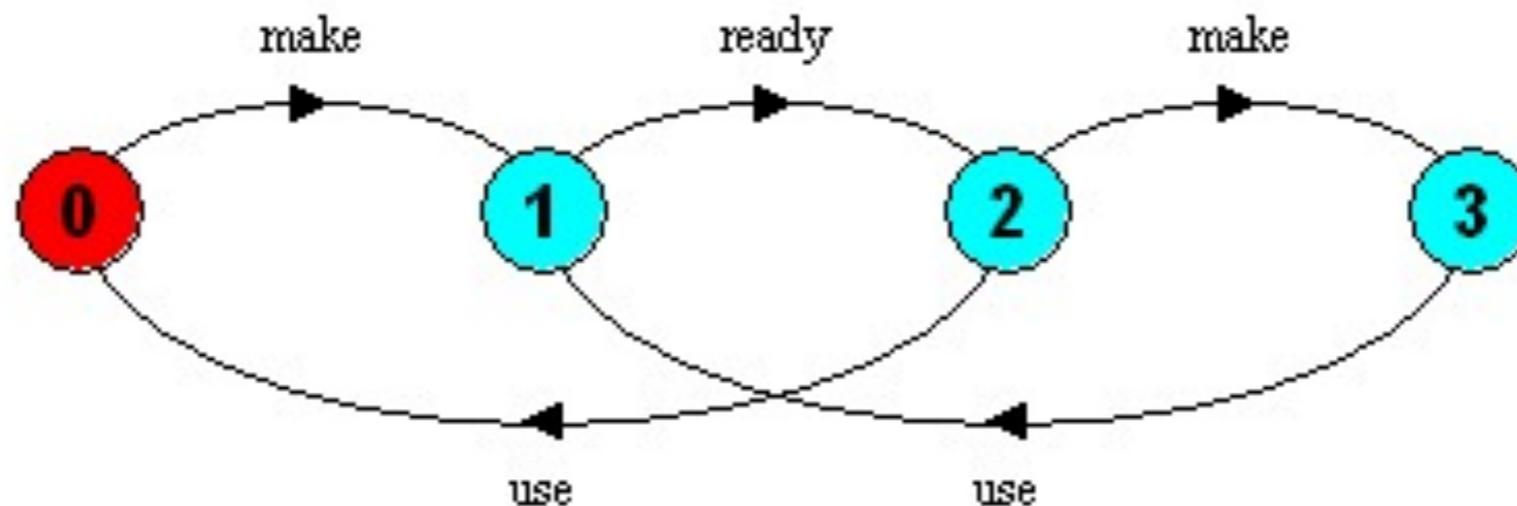
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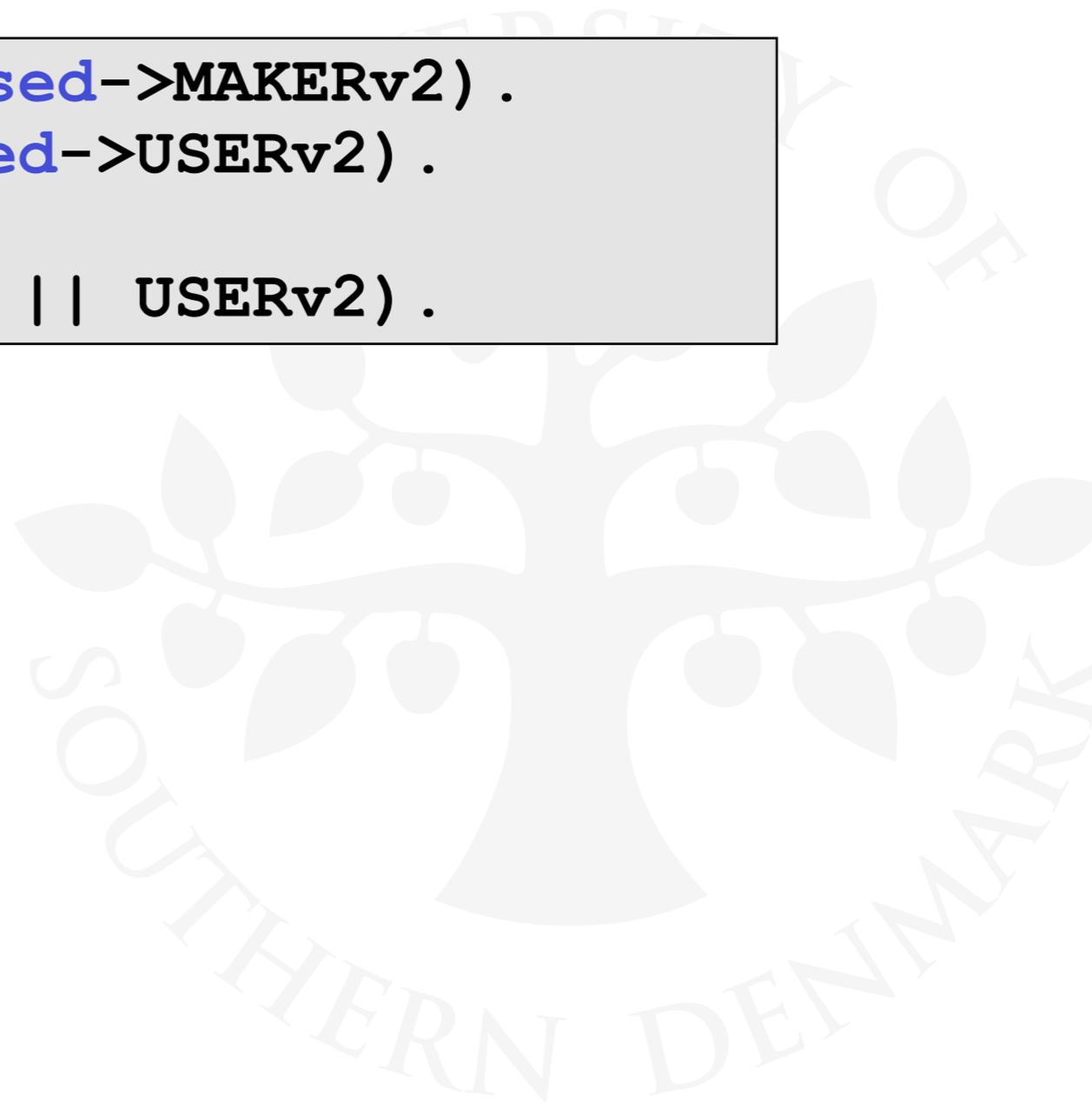
Can we make sure the **MAKER** does not “get ahead of” the **USER** (i.e. never make before use); and if so, how?



Modelling Interaction - Handshake

A handshake is an action acknowledged by another process:

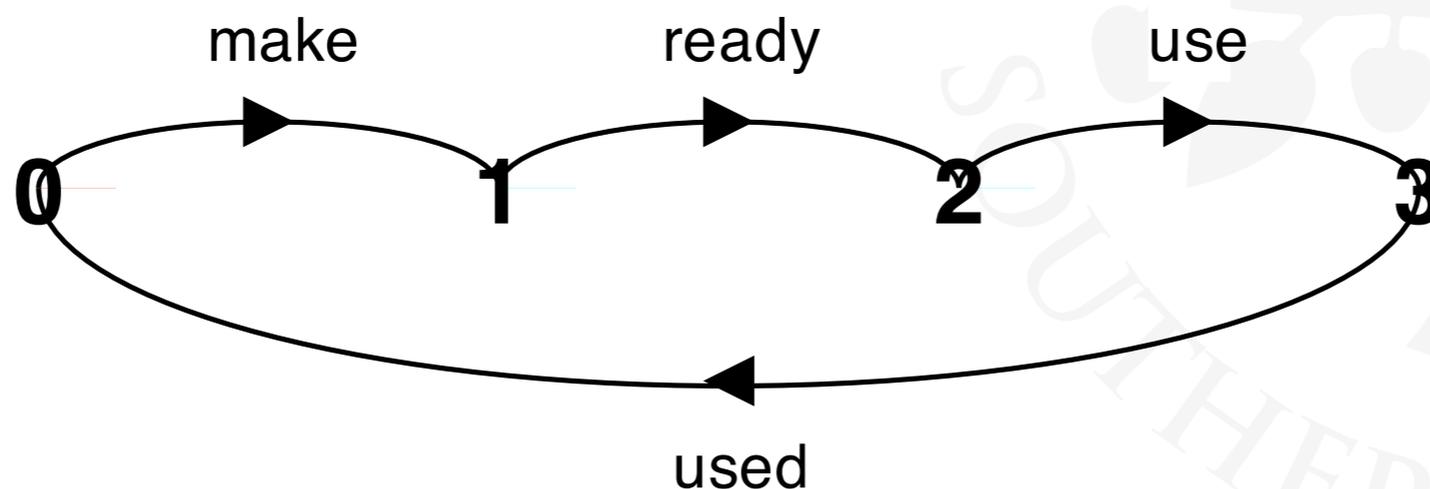
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MAKERv2 = (make->ready->used->MAKERv2) .  
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Modelling Interaction - Multiple Processes

Multi-party synchronisation:

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MAKE_A    = (makeA->ready->used->MAKE_A) .  
MAKE_B    = (makeB->ready->used->MAKE_B) .  
ASSEMBLE  = (ready->assemble->used->ASSEMBLE) .  
  
||FACTORY = (MAKE_A || MAKE_B || ASSEMBLE) .
```

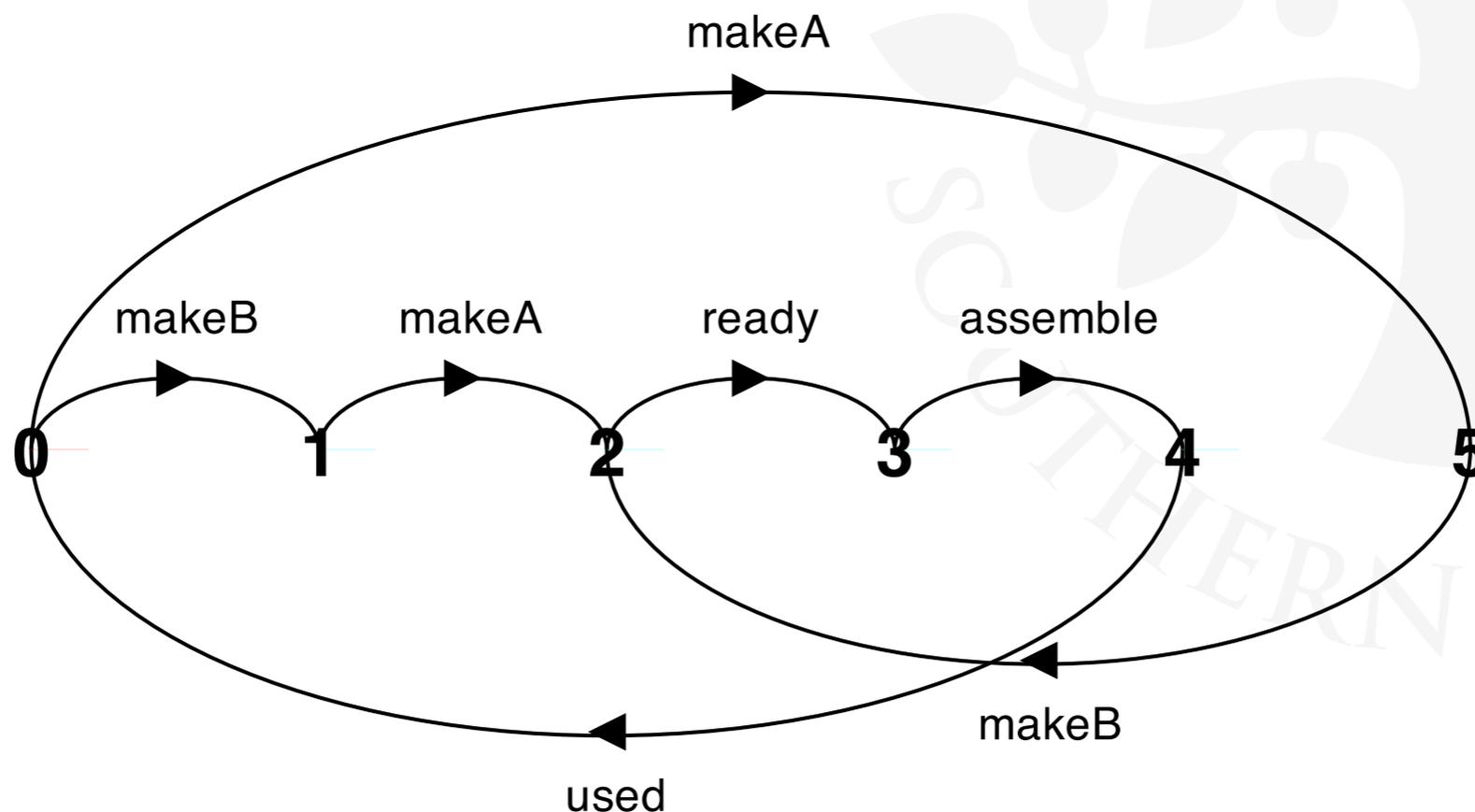




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Composite Processes

A composite process is a parallel composition of primitive processes. These composite processes can be used in the definition of further compositions.

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Further simplification?

↓ associativity!

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Process Labelling

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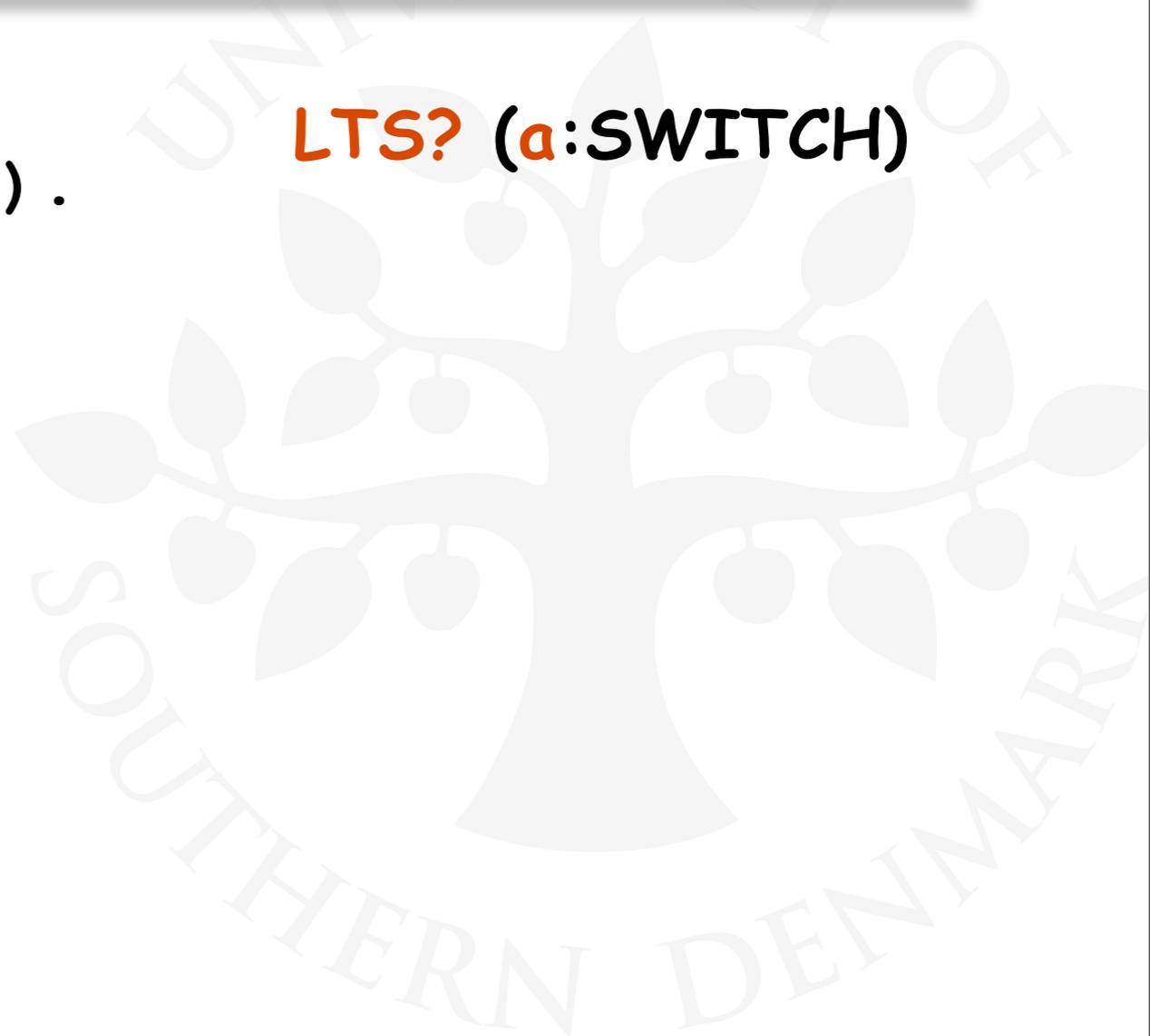
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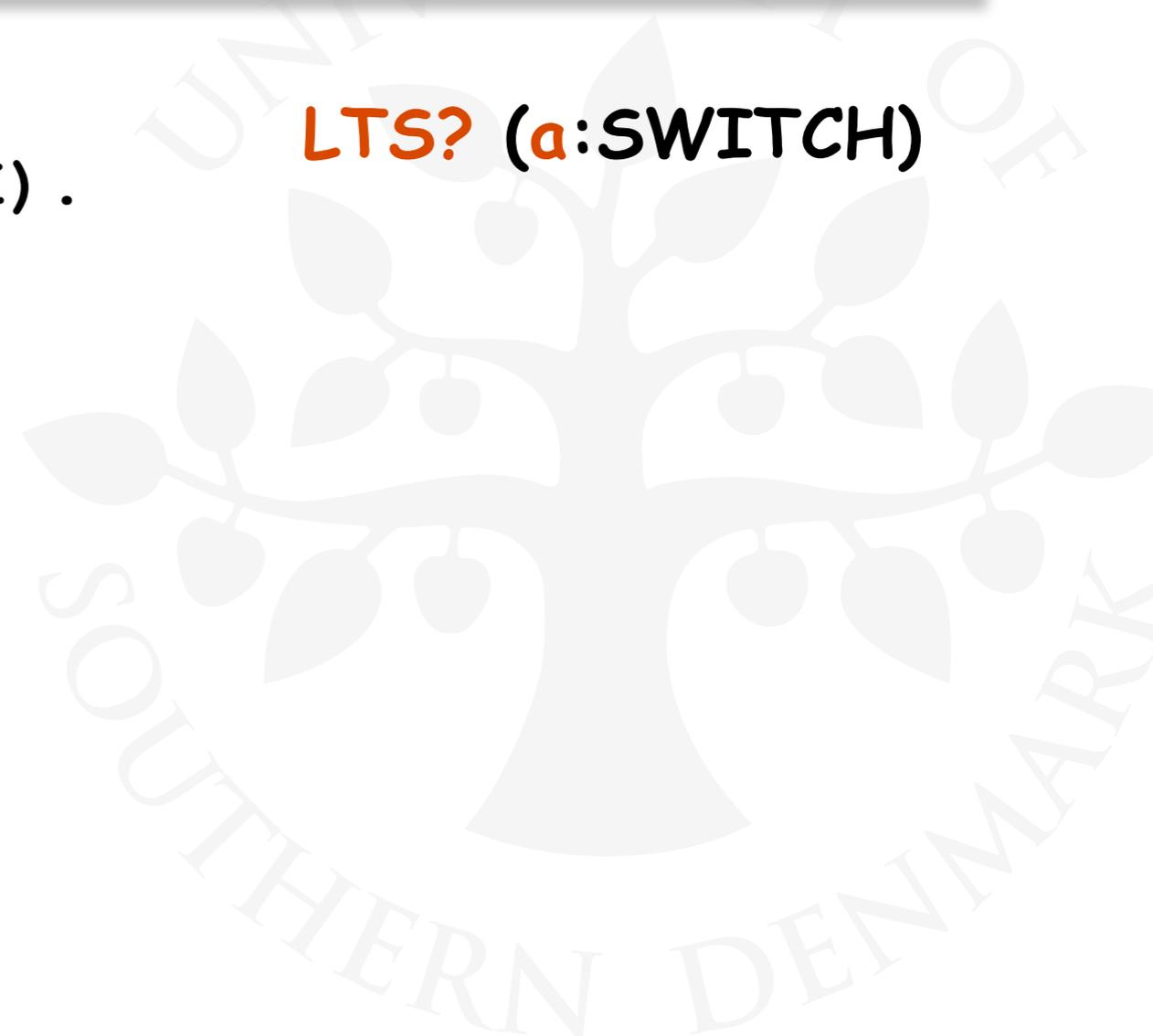
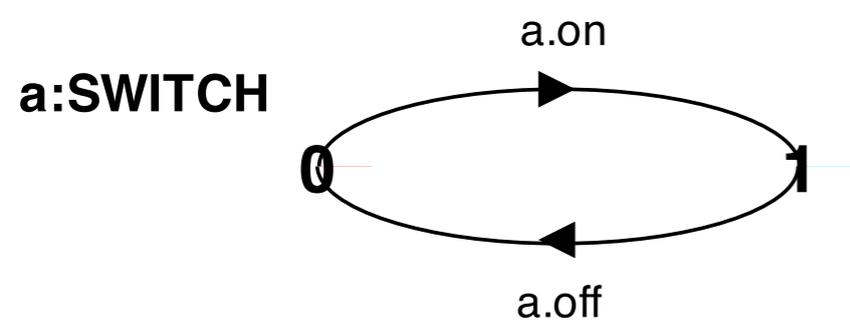
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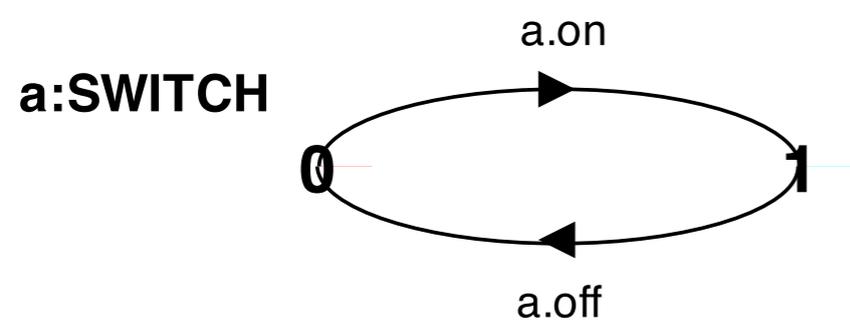
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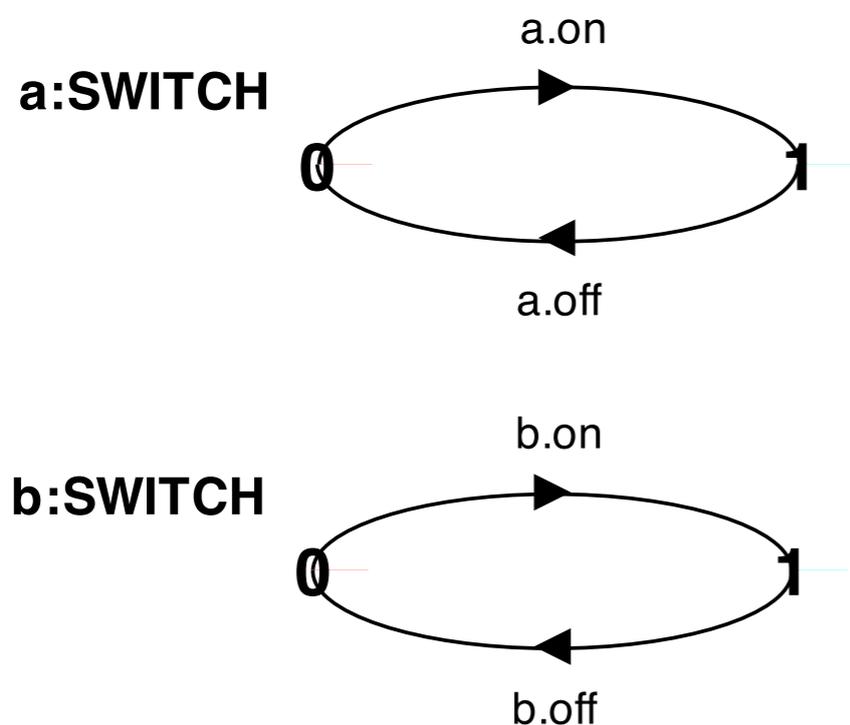
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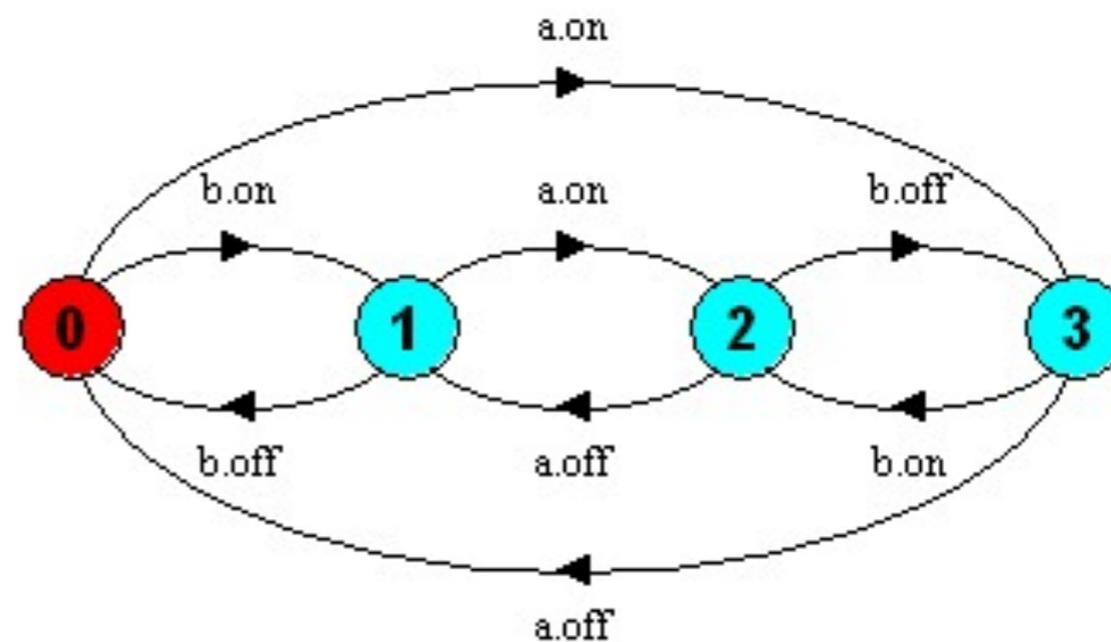
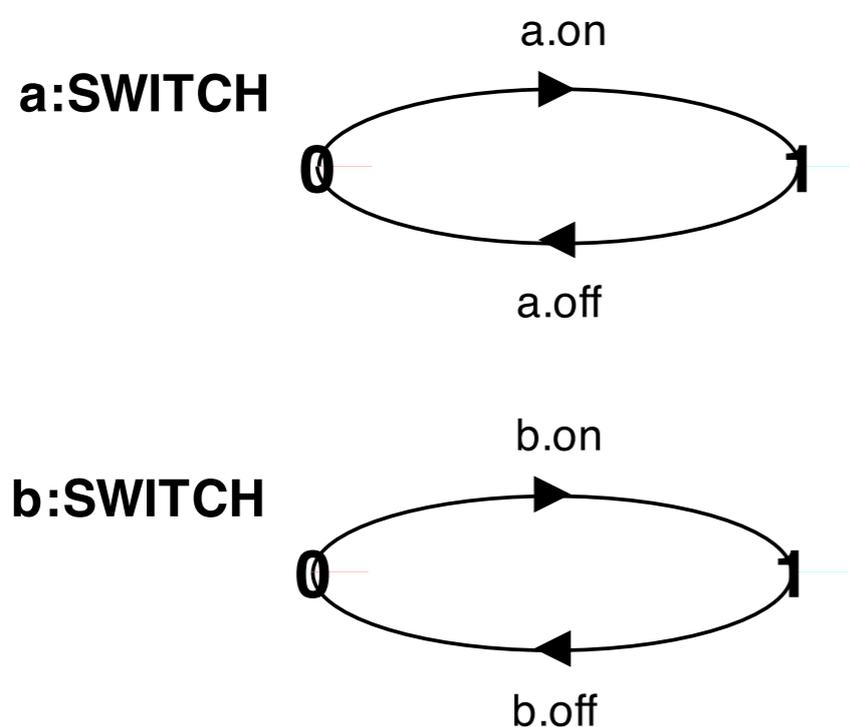
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Process Labelling By A Set Of Prefix Labels



$\{a_1, \dots, a_n\} :: P$ replaces every action label x in the alphabet of P with the labels $a_1.x, \dots, a_n.x$.

Further, every transition $(x \rightarrow X)$ in the definition of P is replaced with the transitions $(\{a_1.x, \dots, a_n.x\} \rightarrow X)$.





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Process prefixing is useful for modelling **shared** resources:

USER = (**acquire** \rightarrow use \rightarrow **release** \rightarrow **USER**) .

RESOURCE = (**acquire** \rightarrow **release** \rightarrow **RESOURCE**) .

RESOURCE_SHARE = (**a** : **USER** || **b** : **USER** || {**a**, **b**} :: **RESOURCE**) .



Process Prefix Labels For Shared Resources

```
RESOURCE = (acquire->release->RESOURCE) .
```

```
USER      = (acquire->use->release->USER) .
```

```
|| RESOURCE_SHARE = (a:USER || b:USER || {a,b}::RESOURCE) .
```



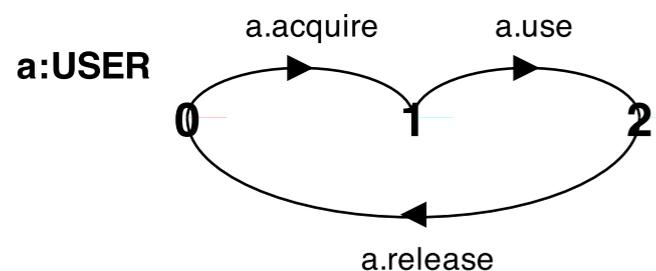


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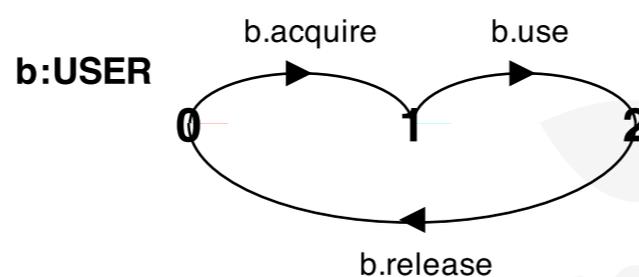
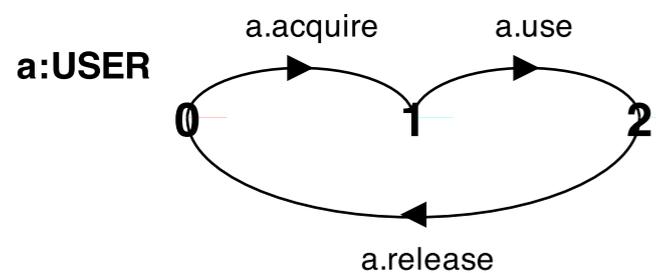


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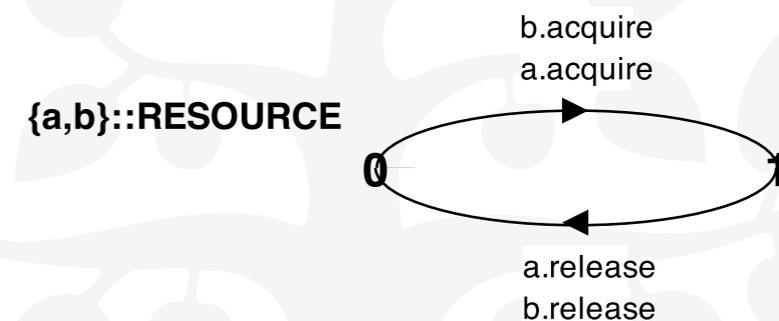
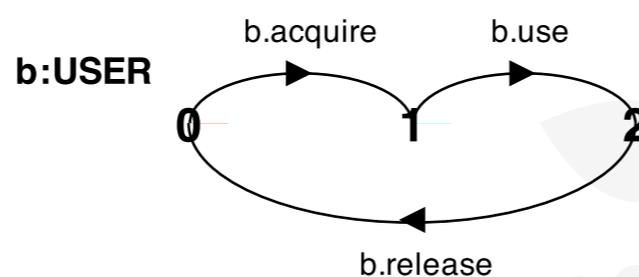
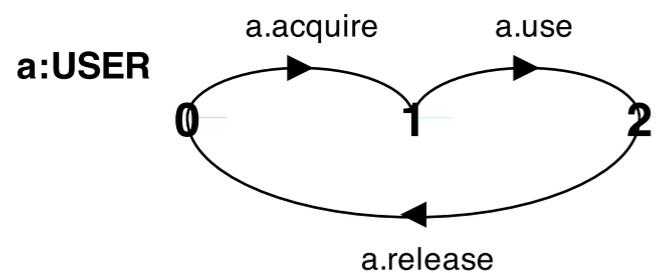


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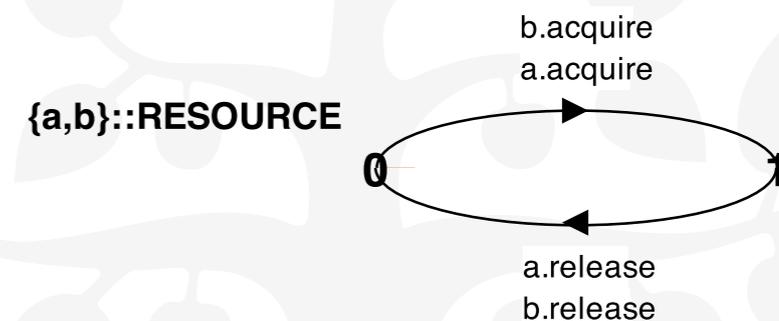
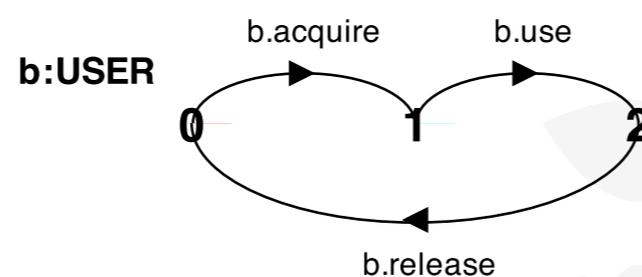
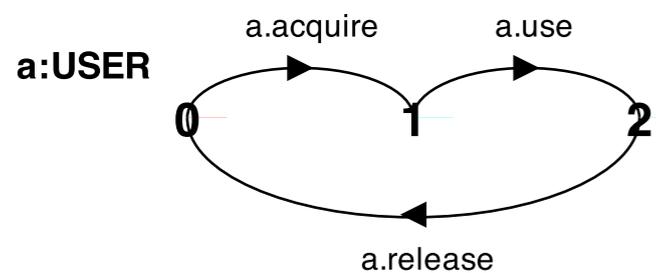


Process Prefix Labels For Shared Resources

RESOURCE = (acquire->release->RESOURCE) .

USER = (acquire->use->release->USER) .

|| RESOURCE_SHARE = (a:USER || b:USER || {a,b}::RESOURCE) .



How does the model ensure that the user that acquires the resource is the one to release it?

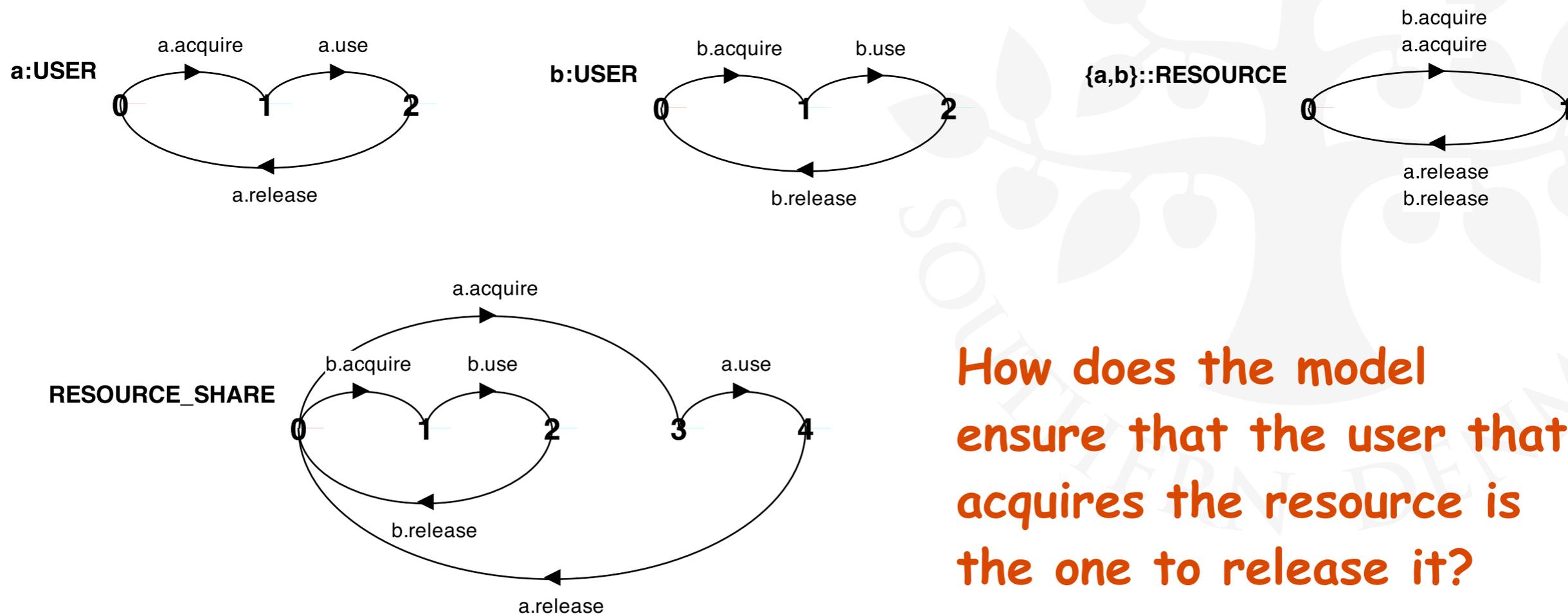


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Example

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x = (x -> STOP) .
```





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LTS? Traces? Number of states?

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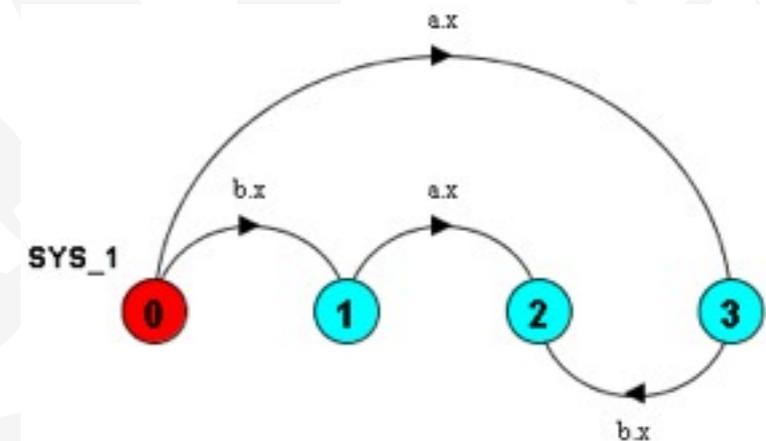
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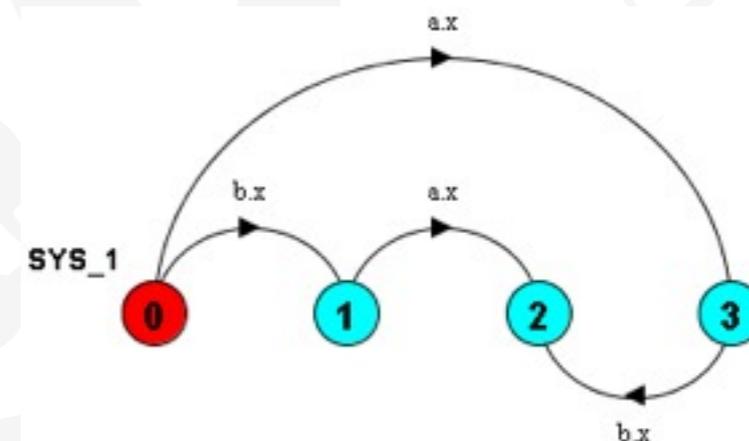


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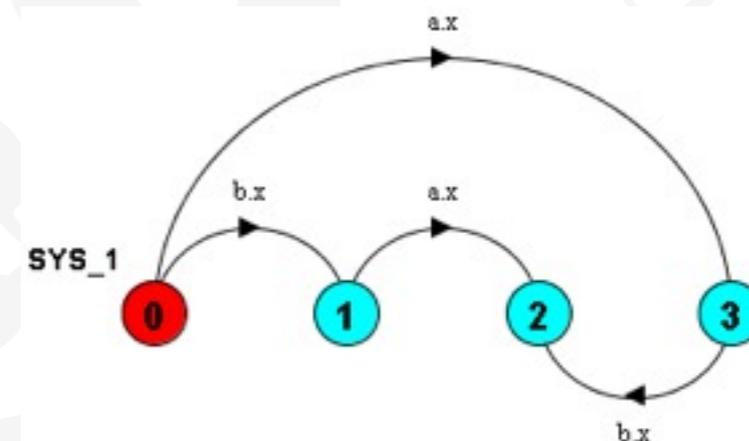
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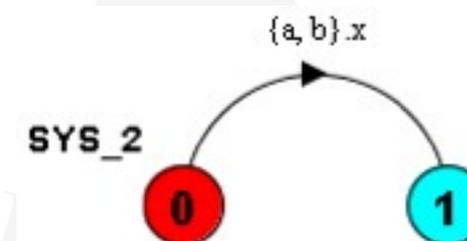
Example

$$X = (x \rightarrow \text{STOP}) .$$
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LTS? Traces? Number of states?


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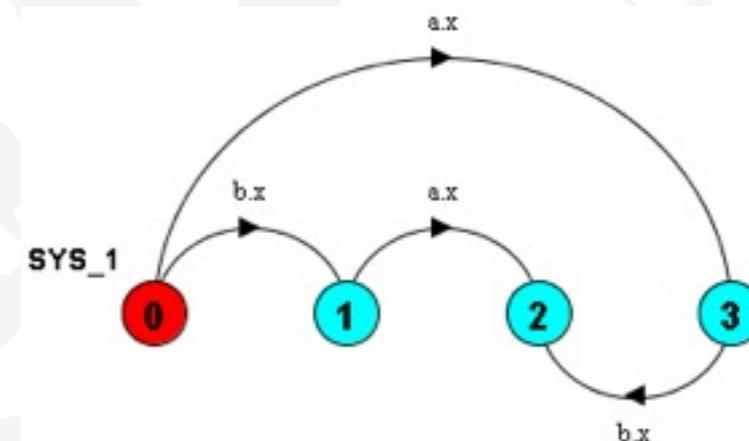


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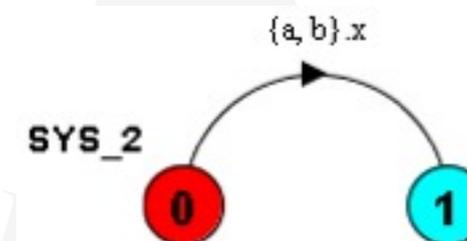
$$|| \text{SYS}_1 = \{a, b\} : X.$$

LTS? Traces? Number of states?
 $\{a \dots\} : X$ creates one process per prefix



$$|| \text{SYS}_2 = \{a, b\} :: X.$$

LTS? Traces? Number of states?
 $\{a \dots\} :: X$ creates one process with all prefixes





Action Relabelling

Relabelling functions are applied to processes to change the names of action labels. The general form of the relabelling function is:

$$\{\text{newlabel}_1/\text{oldlabel}_1, \dots, \text{newlabel}_n/\text{oldlabel}_n\}.$$




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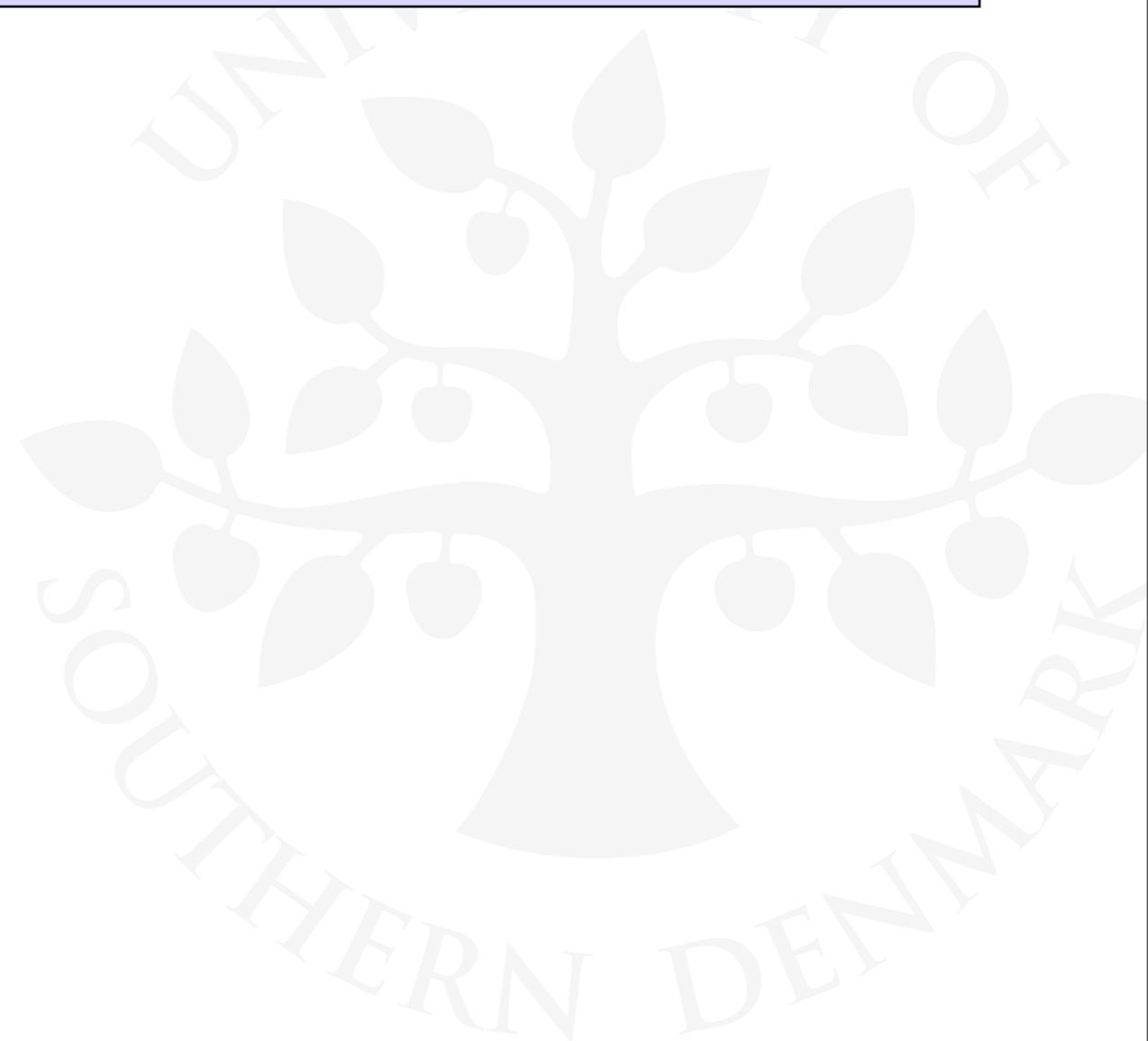
```
SERVER = (request->service->reply->SERVER) .
```



Action Relabelling

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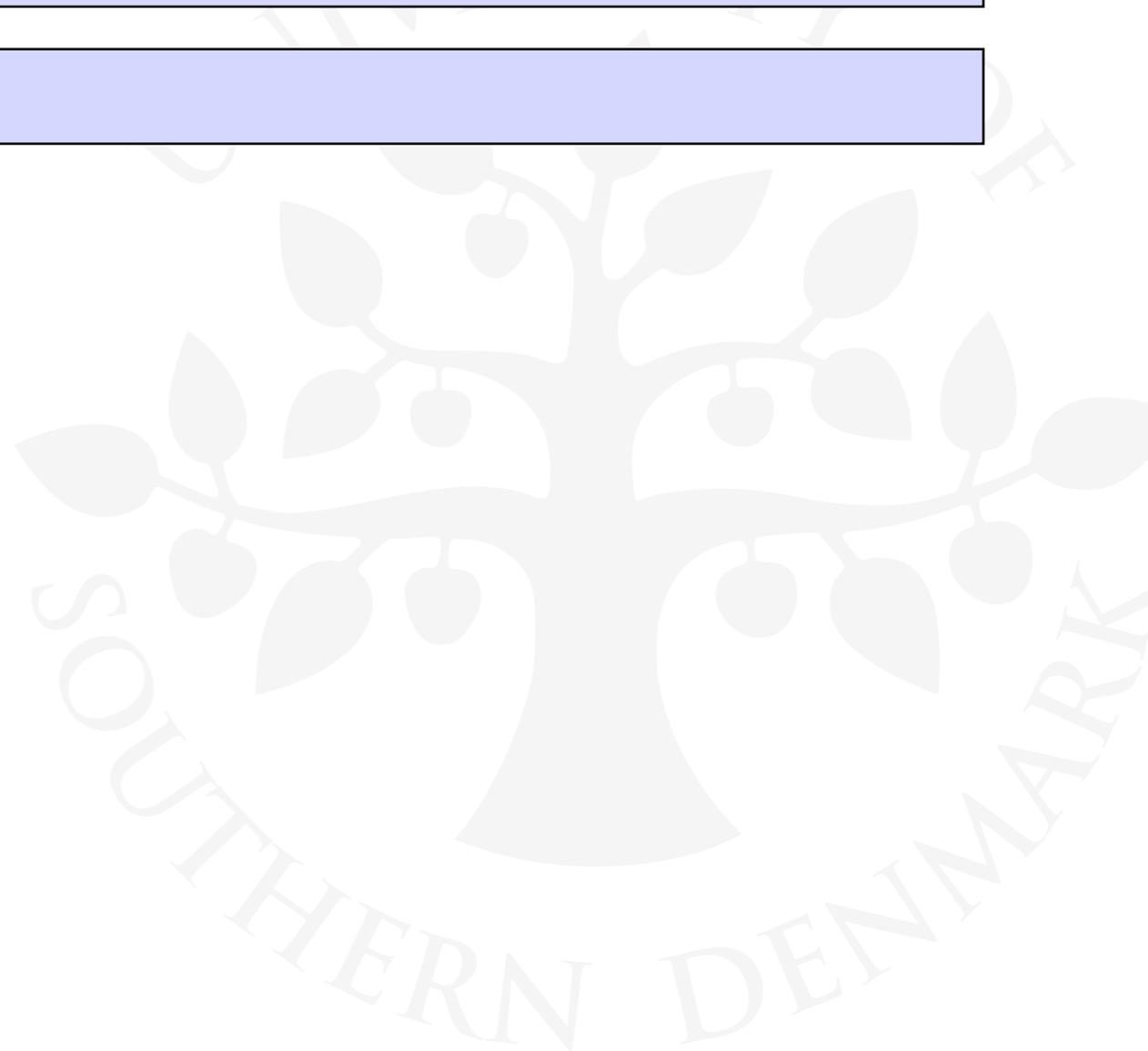


Action Relabelling

```
CLIENT = (call->wait->continue->CLIENT) .
```

```
SERVER = (request->service->reply->SERVER) .
```

```
C = (CLIENT /{reply/wait}) .
```





Action Relabelling

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Action Relabelling

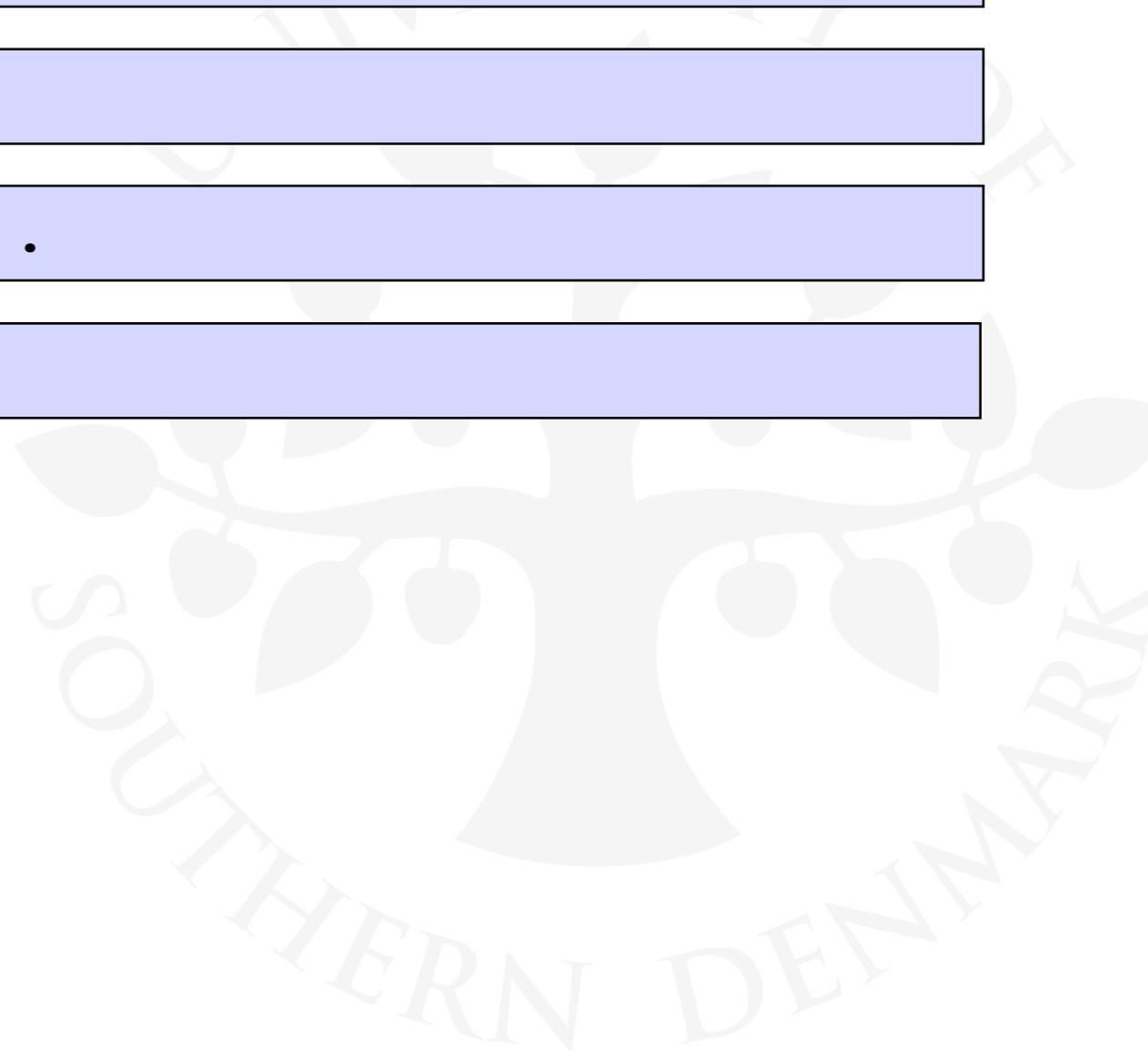
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Action Relabelling

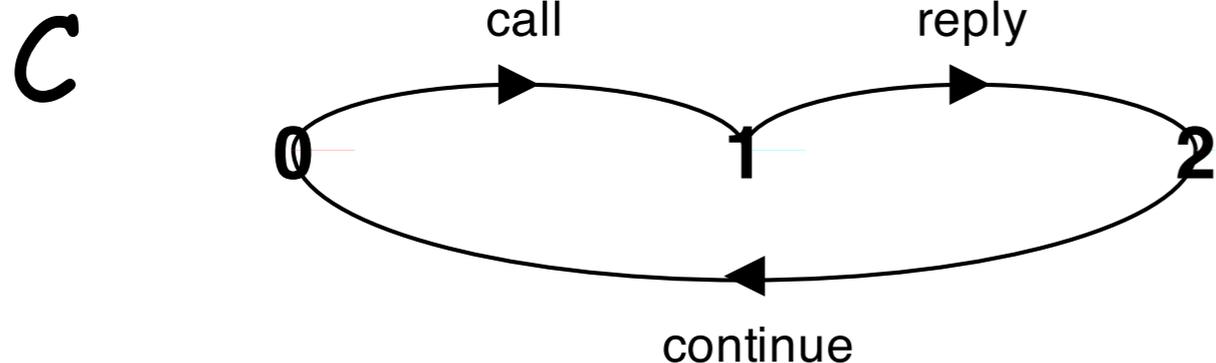
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Action Relabelling

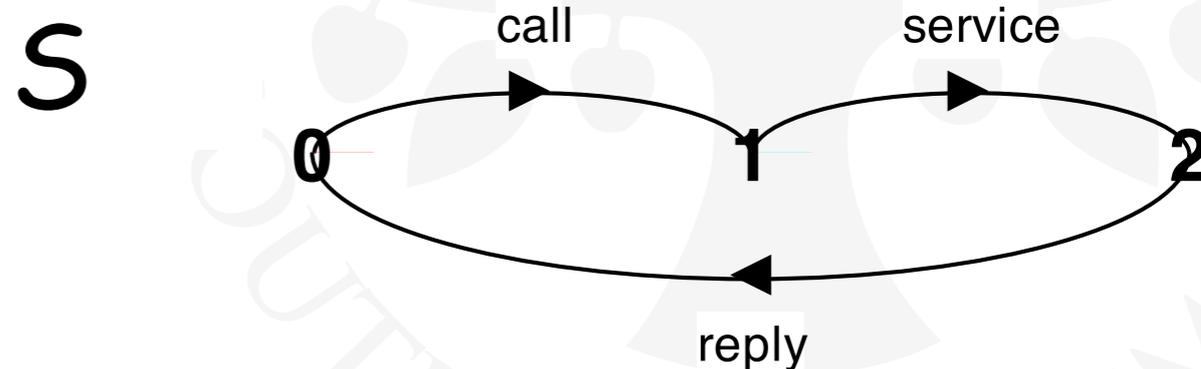
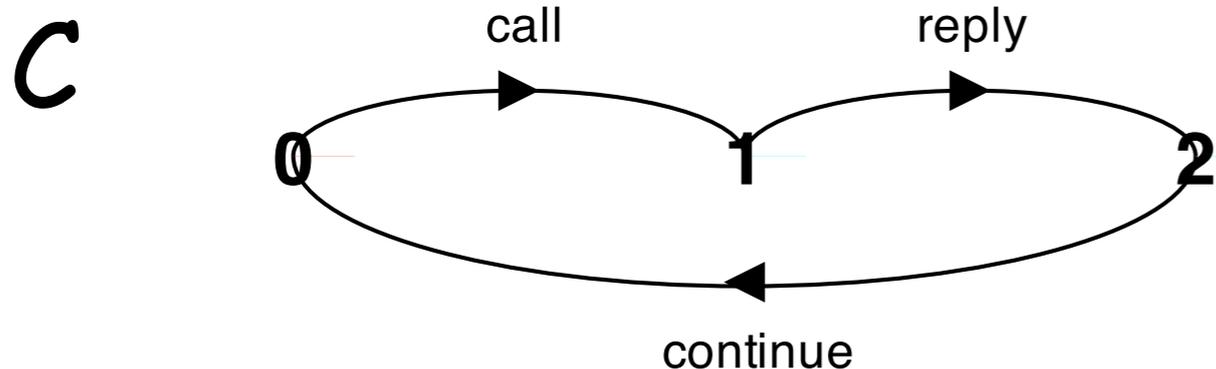
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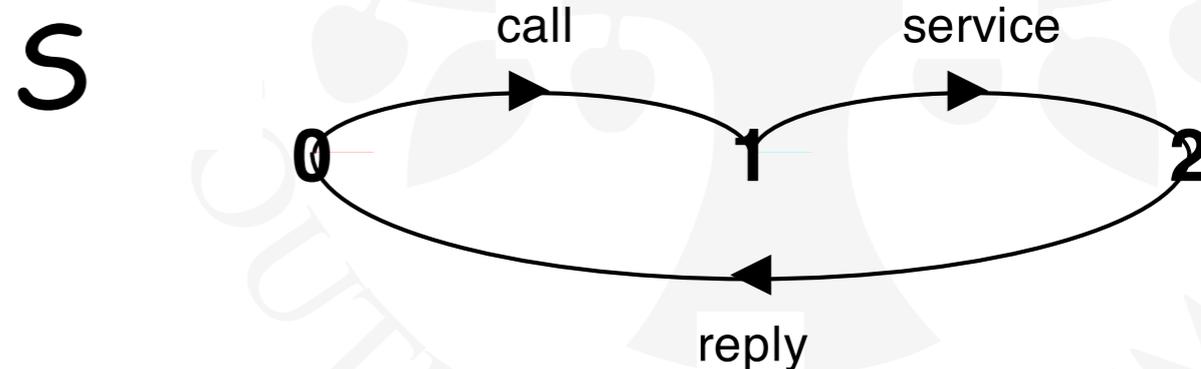
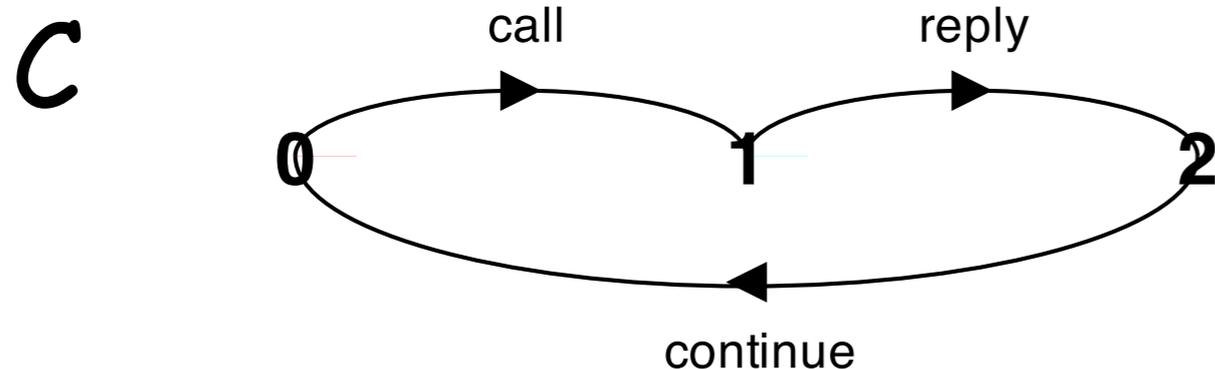
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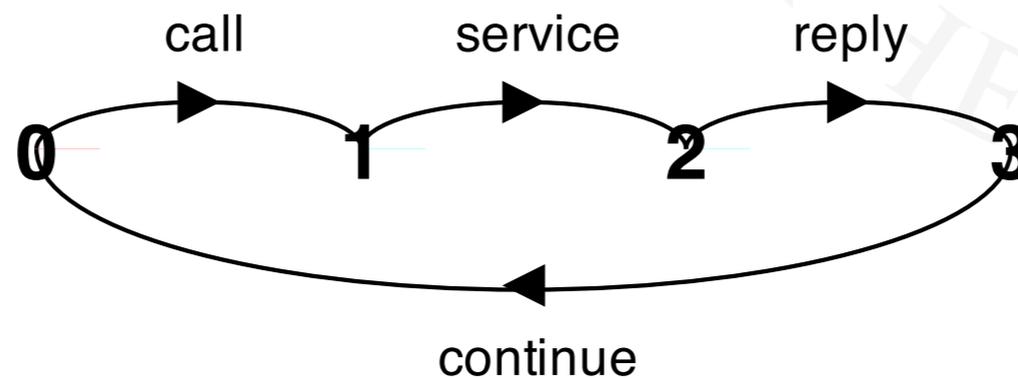
```
C = (CLIENT /{reply/wait}).
```

```
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```
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```



C_S





Action Relabelling - Prefix Labels

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||CLIENT_SERVERv2 = (CLIENTv2 || SERVERv2)
                    /{call/accept} .
```

When applied to a process P , the hiding operator $\backslash\{a_1, \dots, a_x\}$ removes the action names $a_1..a_x$ from the alphabet of P and makes these concealed actions "silent".

These silent actions are labelled **tau**.

Silent actions in different processes are not shared.



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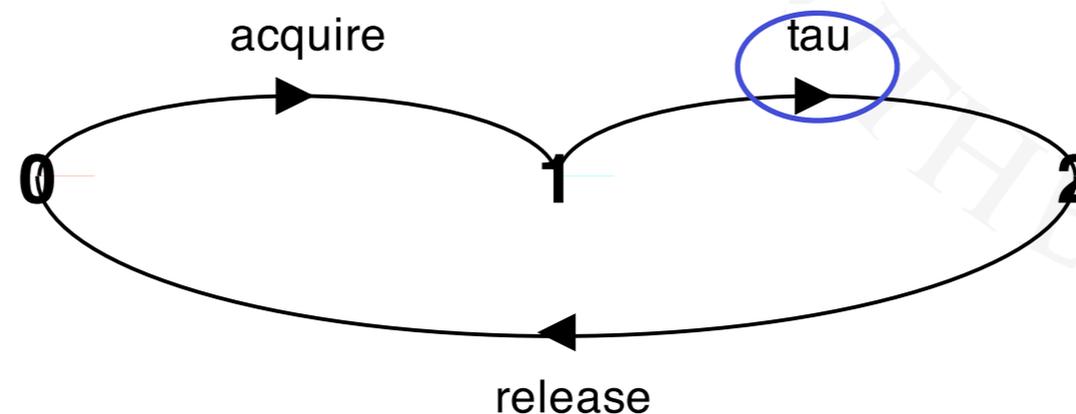
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USER = (acquire->use->release->USER)
      \{use}.
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$USER = (acquire \rightarrow use \rightarrow release \rightarrow USER) \backslash\{use\}.$



Action Hiding - Abstraction To Reduce Complexity

Sometimes it is more convenient to specify the set of labels to be **exposed**....

When applied to a process P , the interface operator $@\{a_1, \dots, a_x\}$ hides all actions in the alphabet of P not labelled in the set $a_1..a_x$.

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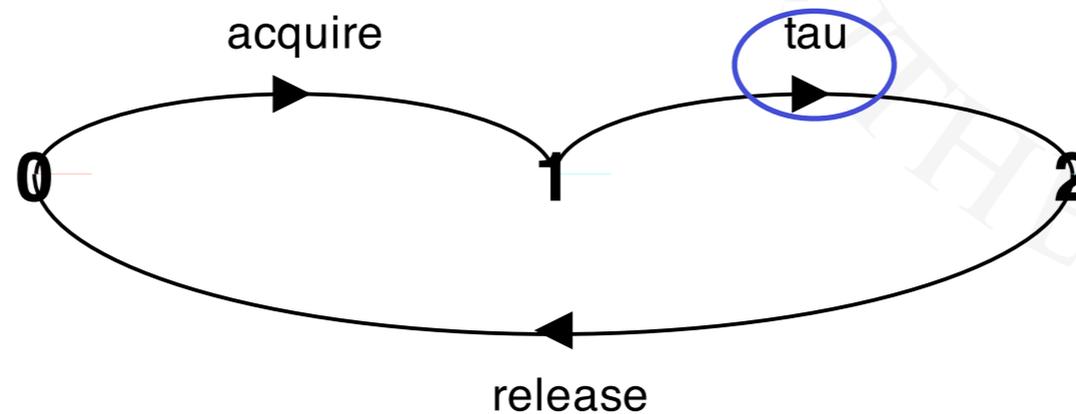
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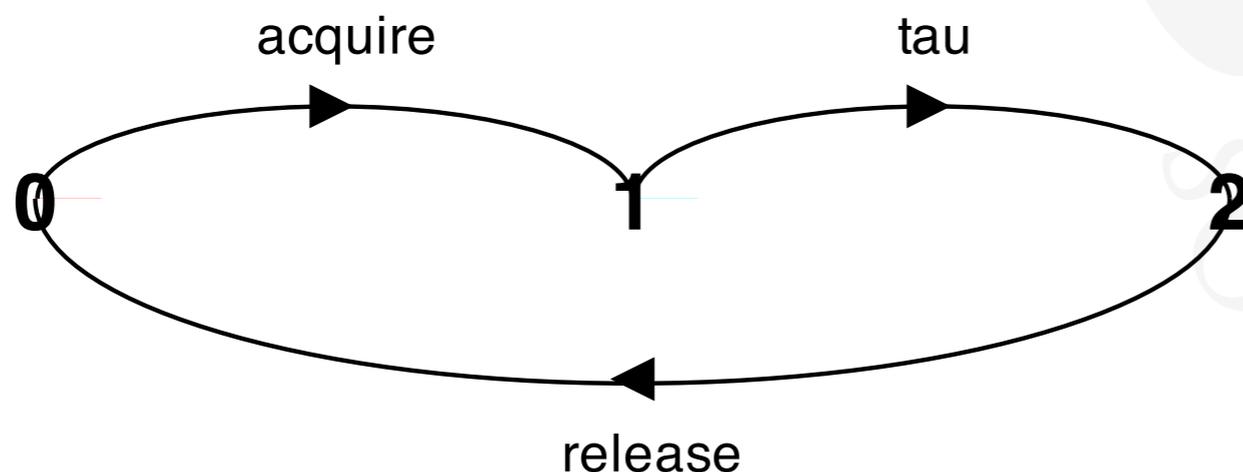


Action Hiding

The following definitions are equivalent:

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```

```
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```

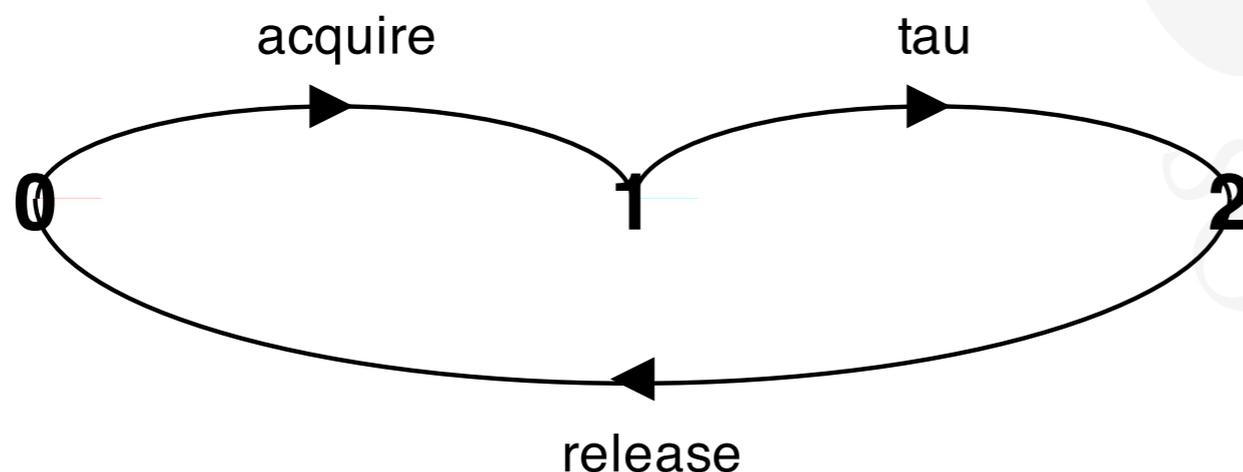


Action Hiding

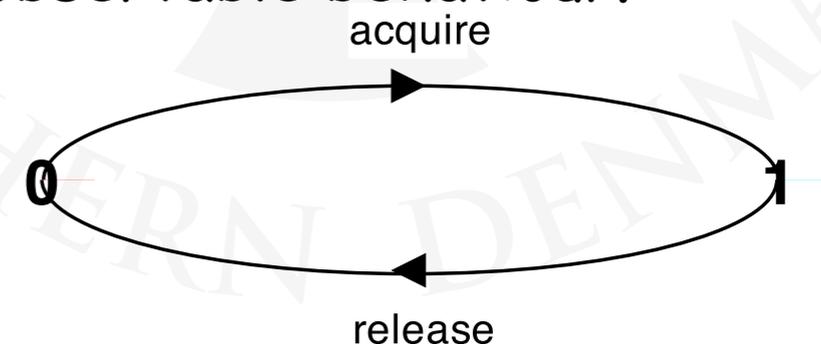
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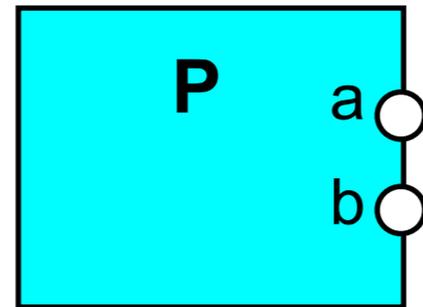
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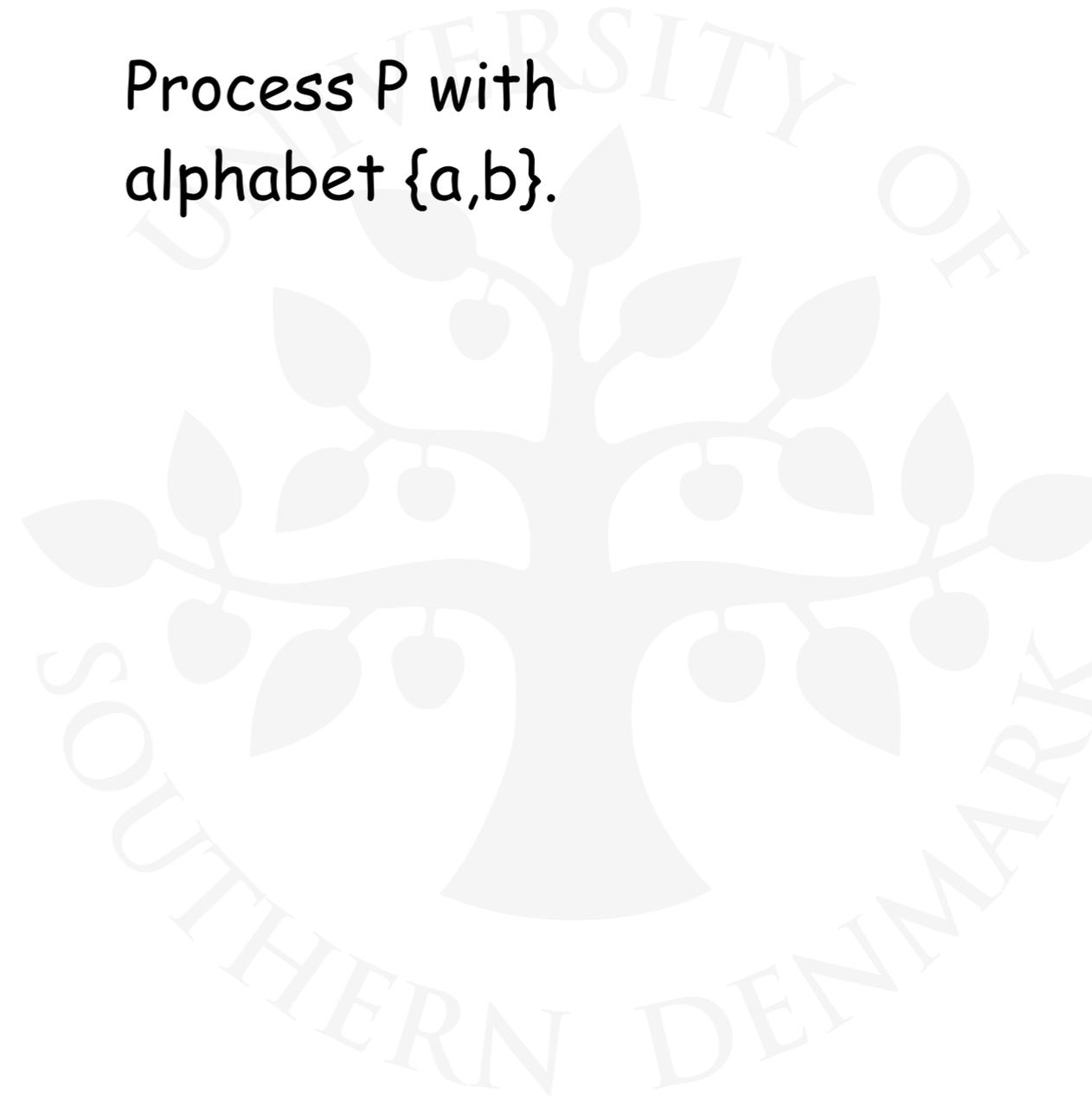
Minimisation removes hidden tau actions to produce an LTS with equivalent observable behaviour.



Structure Diagrams

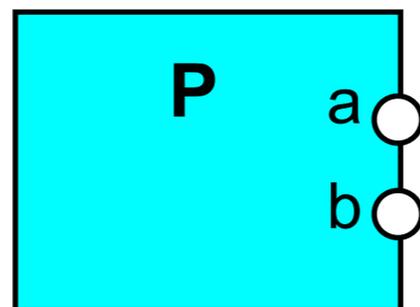


Process P with
alphabet {a,b}.





Structure Diagrams

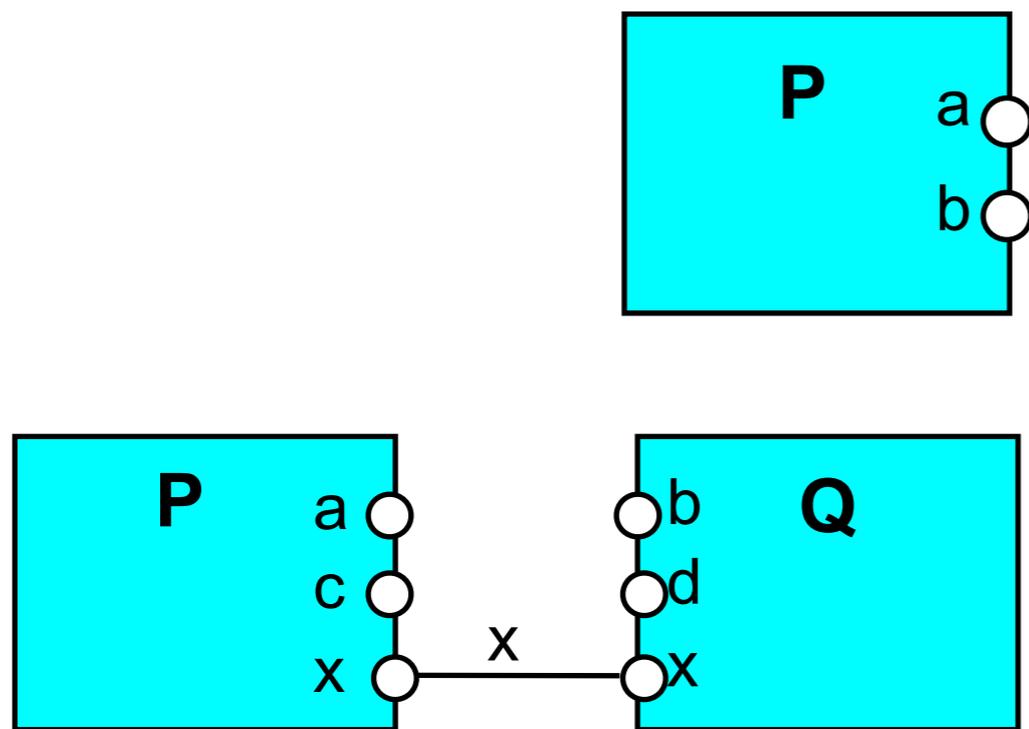


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Parallel Composition
(P||Q)



Structure Diagrams

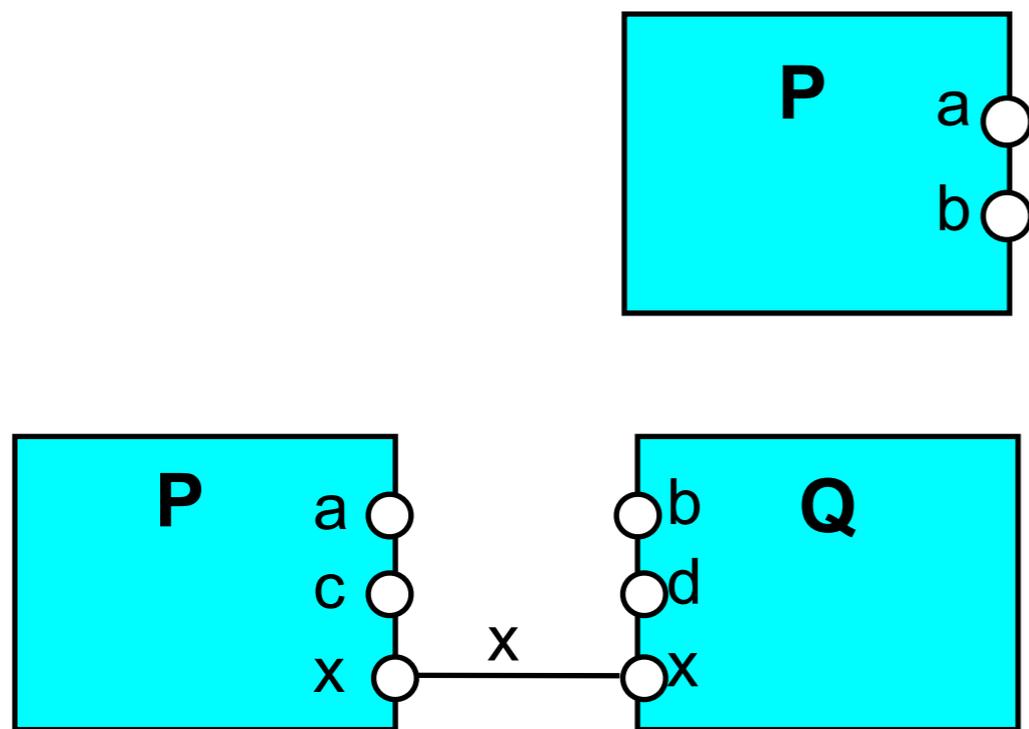


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Structure Diagrams

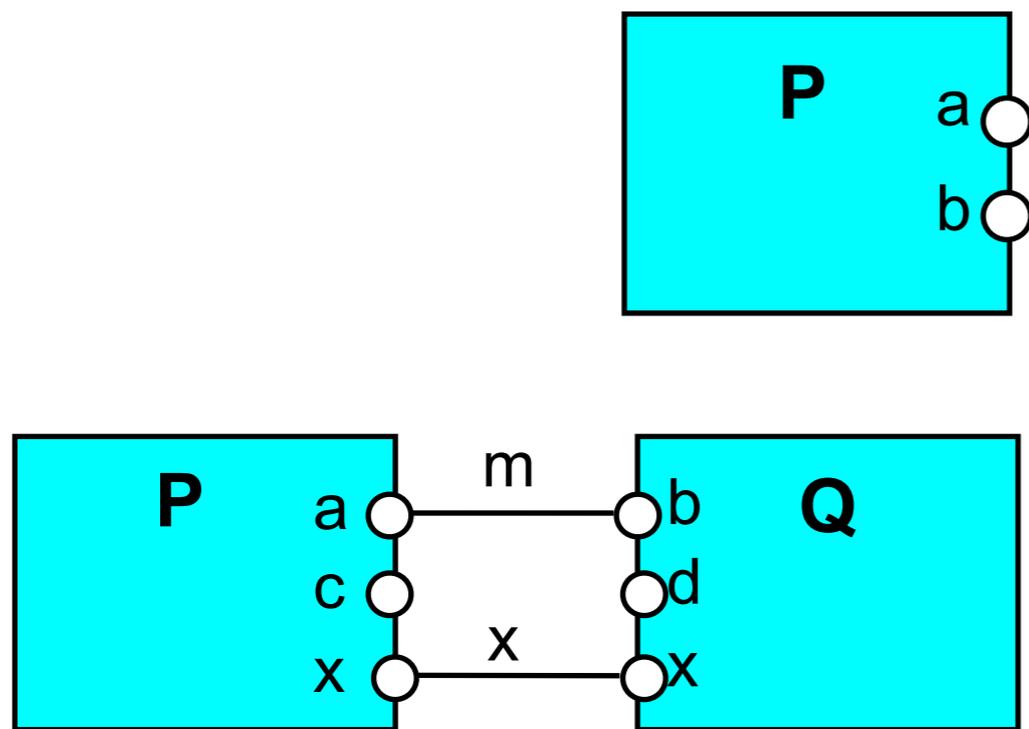


Process *P* with
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Parallel Composition
 $(P \parallel Q) / \{m/a, m/b\}$



Structure Diagrams

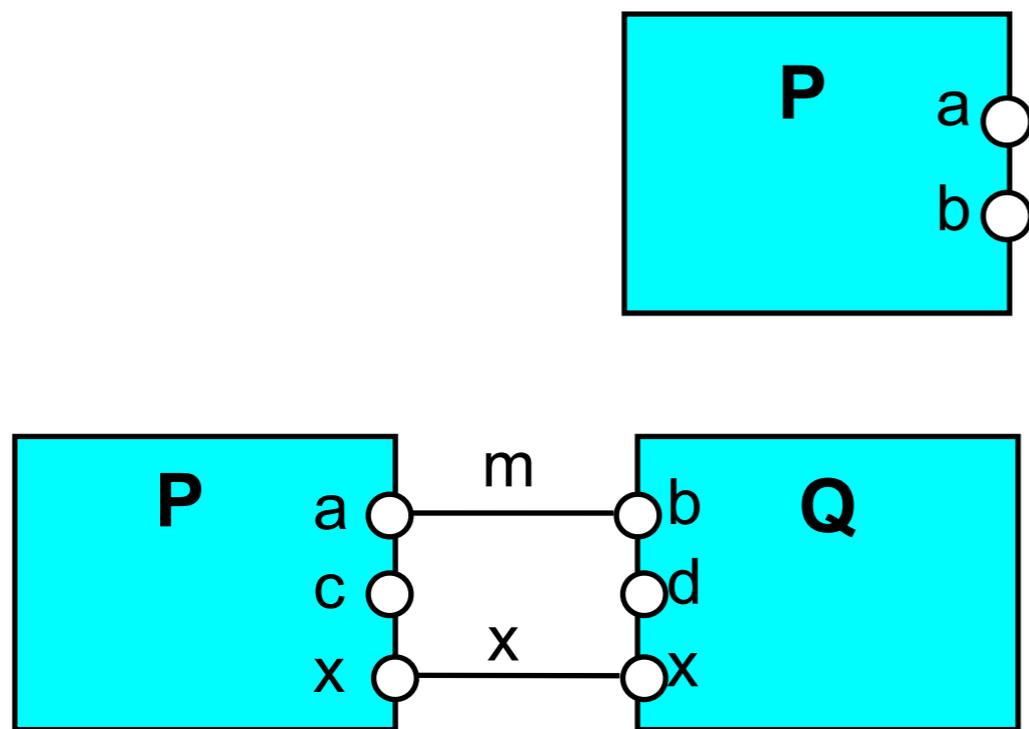


Process **P** with
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Structure Diagrams

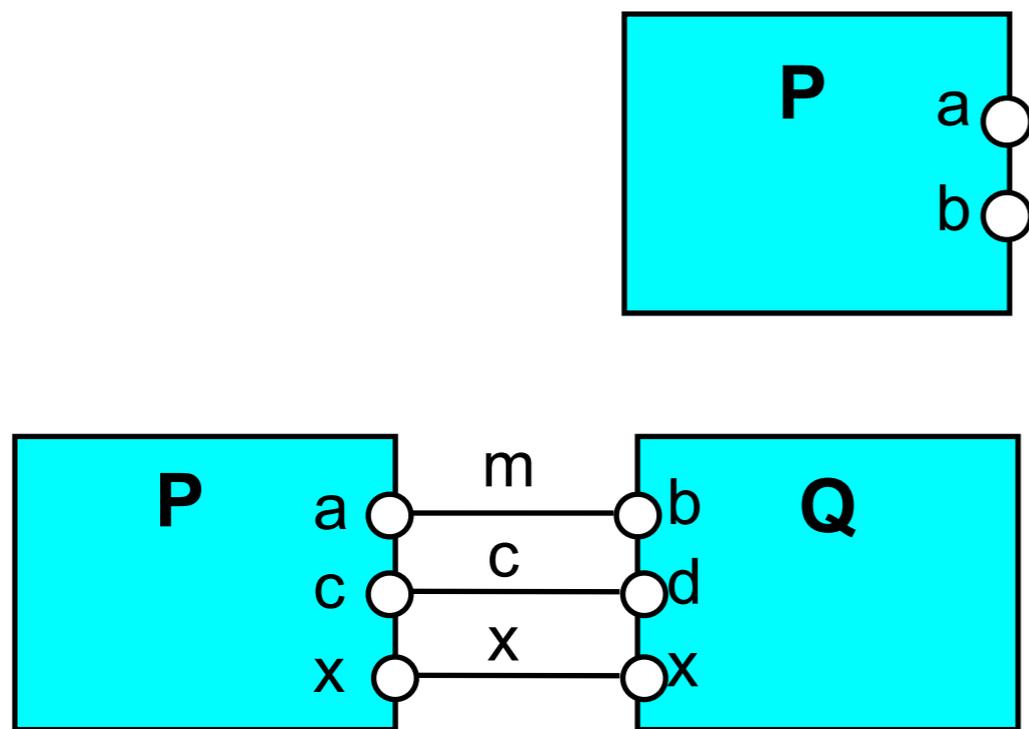


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Structure Diagrams

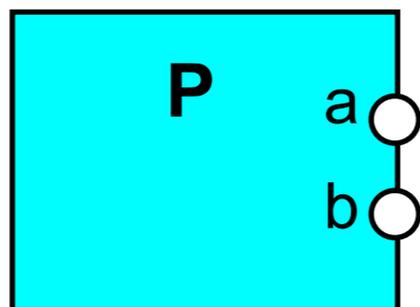


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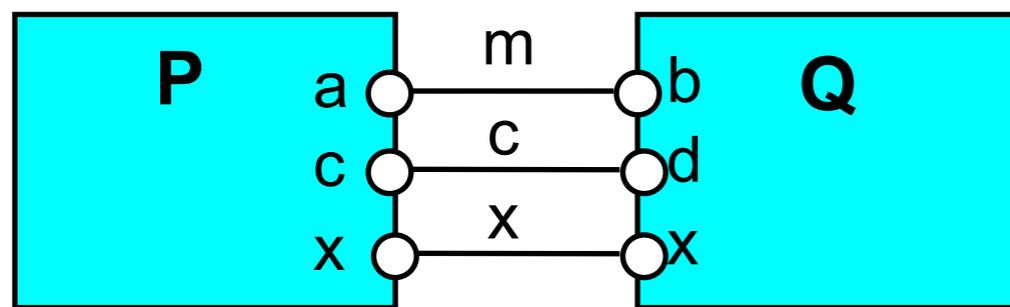
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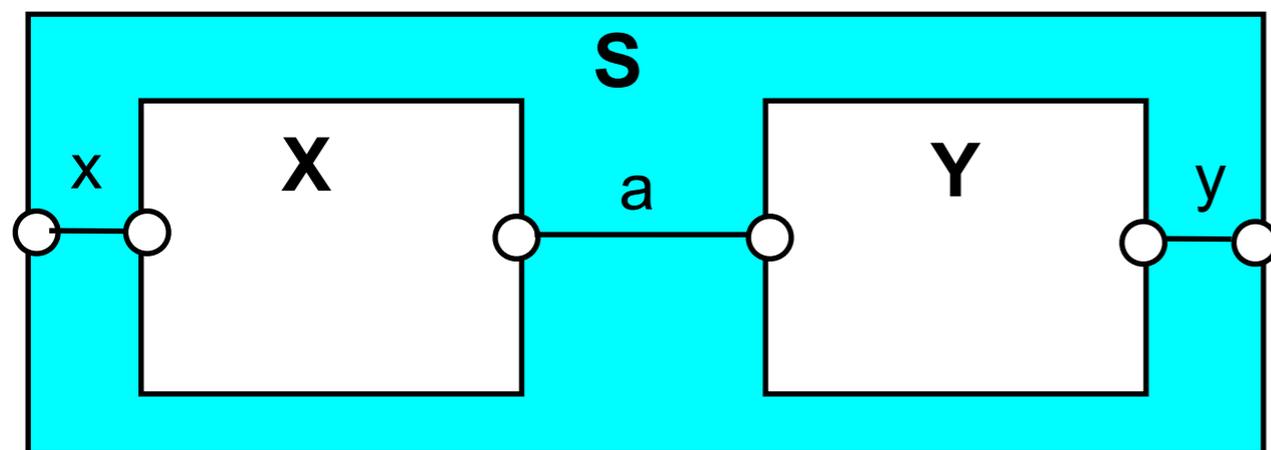
Structure Diagrams



Process P with alphabet {a,b}.



Parallel Composition $(P || Q) / \{m/a, m/b, c/d\}$



Composite process $||S = (X || Y) @ \{x,y\}$



Structure Diagrams

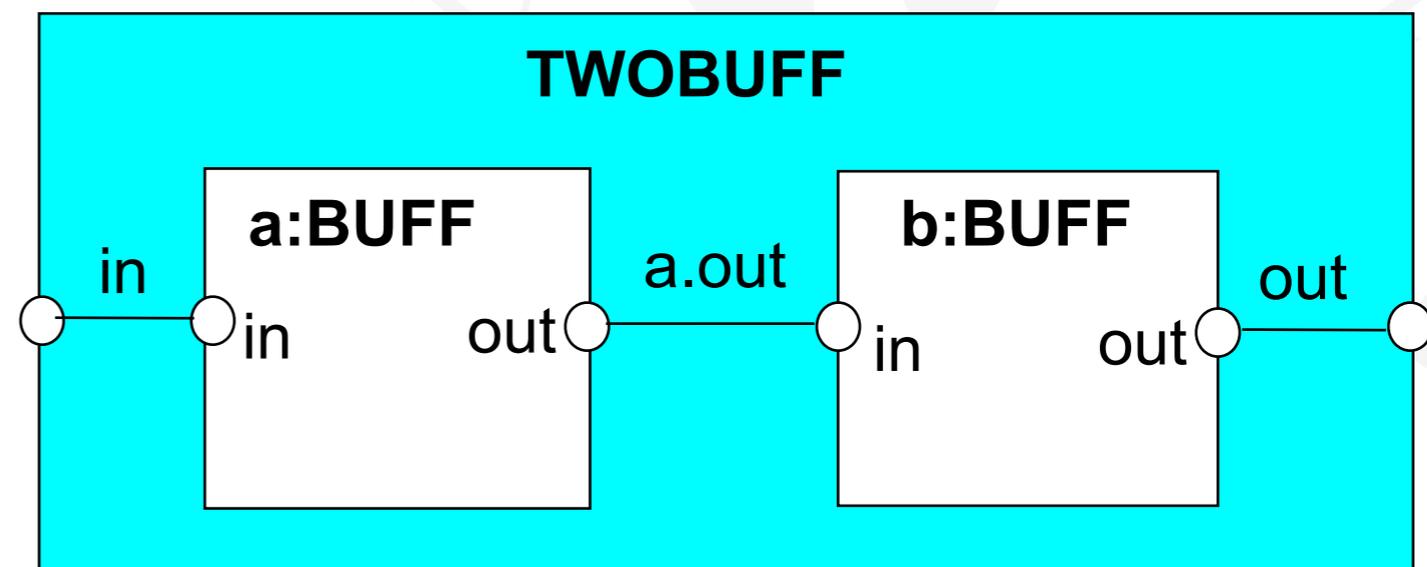
```
range T = 0..3  
BUFF = (in[i:T]->out[i]->BUFF) .
```



Structure Diagrams

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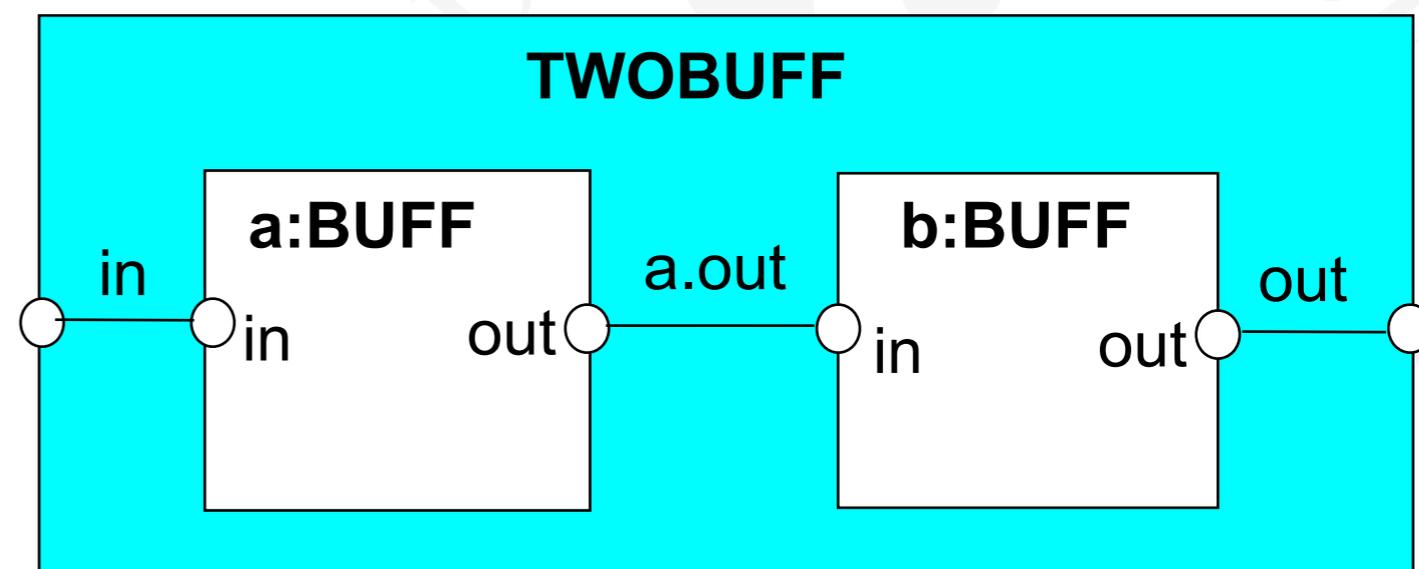
We use structure diagrams to capture the structure of a model expressed by the static combinators: parallel composition, relabelling and hiding.



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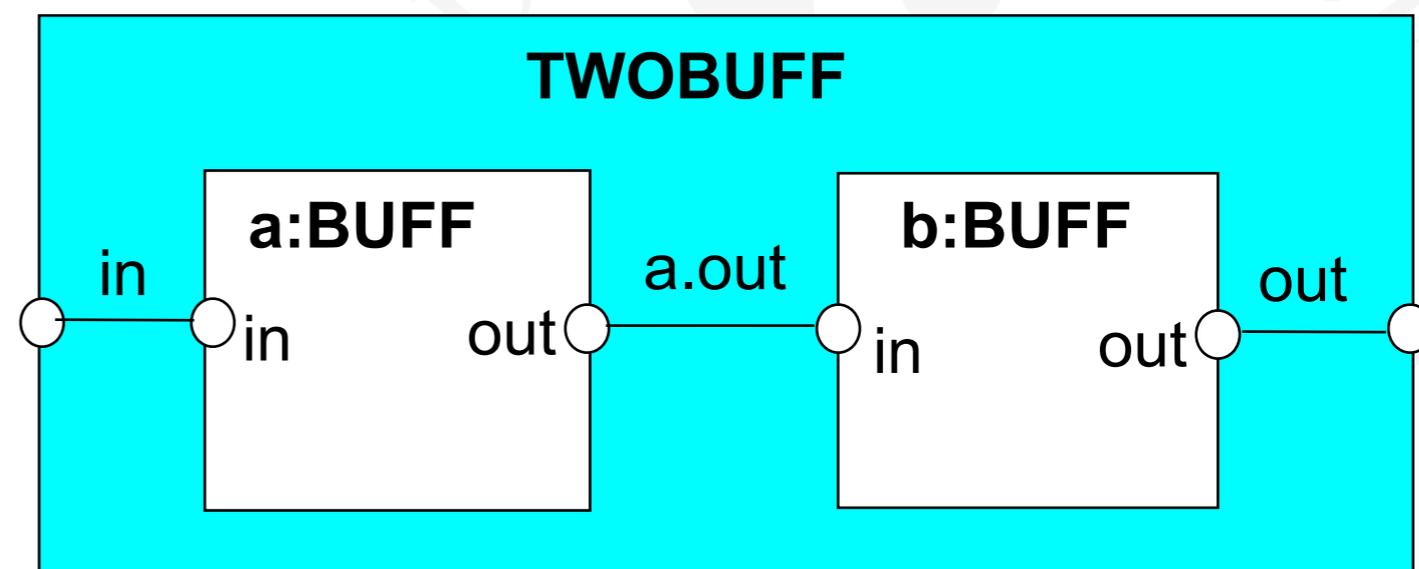


```
|| TWOBUFF =
```

Structure Diagrams

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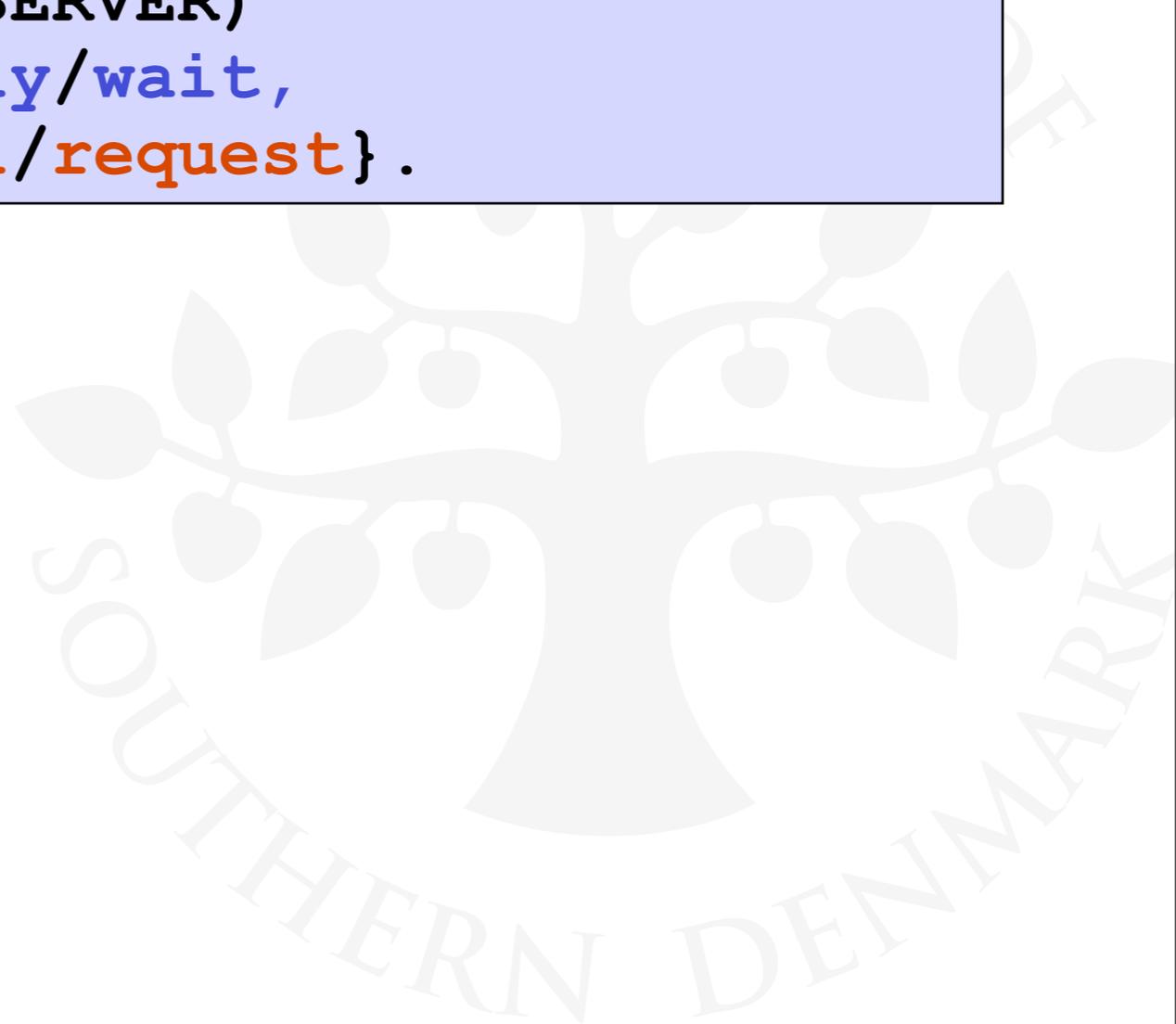


```
|| TWOBUFF = (a:BUFF || b:BUFF)  
          /{in/a.in, a.out/b.in, out/b.out}  
          @{in,out} .
```



Structure Diagrams

```
CLIENT = (call->wait->continue->CLIENT) .  
SERVER = (request->service->reply->SERVER) .  
  
||CLIENT_SERVER = (CLIENT||SERVER)  
                    /{reply/wait,  
                     call/request} .
```





Structure Diagrams

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CLIENT = (call->wait->continue->CLIENT) .  
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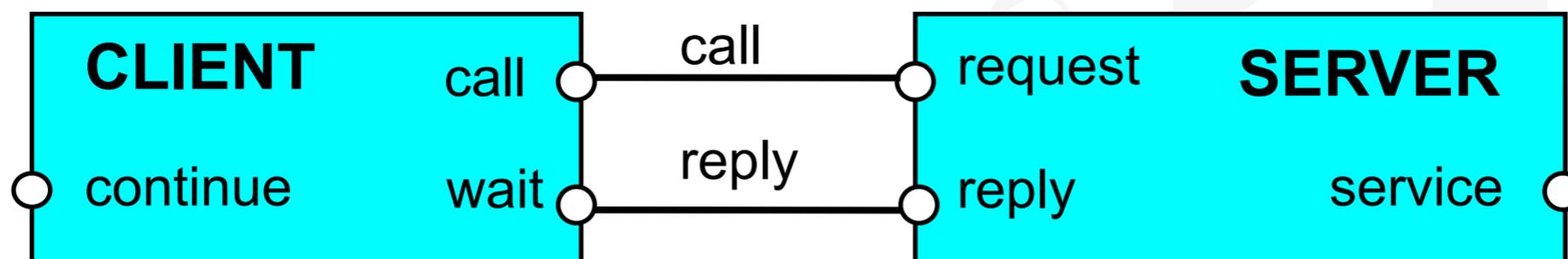
Structure diagram for CLIENT_SERVER ?



Structure Diagrams

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Structure diagram for CLIENT_SERVER ?





Structure Diagrams

```
SERVERv2 = (accept.request  
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```





Structure Diagrams

```
SERVERv2 = (accept.request  
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                    /{call/accept} .
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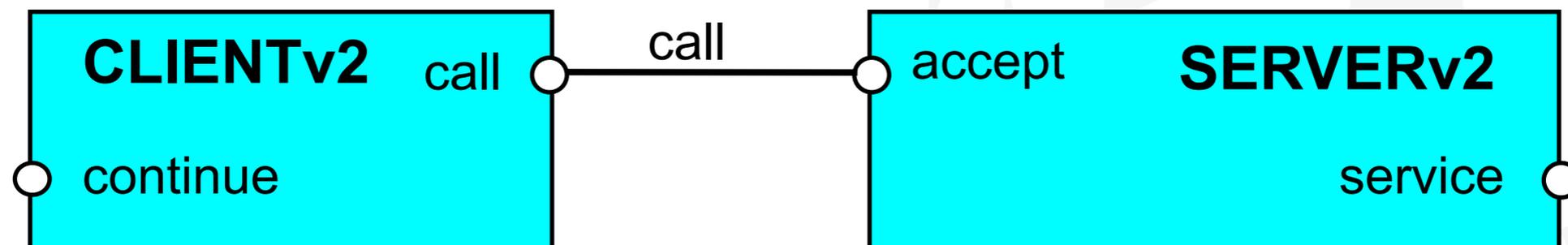
Structure diagram for CLIENT_SERVERv2 ?



Structure Diagrams

```
SERVERv2 = (accept.request  
            ->service->accept.reply->SERVERv2) .  
CLIENTv2 = (call.request  
            ->call.reply->continue->CLIENTv2) .  
||CLIENT_SERVERv2 = (CLIENTv2 || SERVERv2)  
                    /{call/accept} .
```

Structure diagram for CLIENT_SERVERv2 ?



Simply use the shared prefix.

Structure Diagrams - Resource Sharing





Structure Diagrams - Resource Sharing

```
RESOURCE = (acquire->release->RESOURCE) .
```





Structure Diagrams - Resource Sharing

```
RESOURCE = (acquire->release->RESOURCE) .  
USER      = (printer.acquire->use->printer.release->USER) .
```





Structure Diagrams - Resource Sharing

```
RESOURCE = (acquire->release->RESOURCE) .  
USER      = (printer.acquire->use->printer.release->USER) .
```

```
|| PRINTER_SHARE =  
  (a:USER || b:USER || {a,b}::printer:RESOURCE) .
```



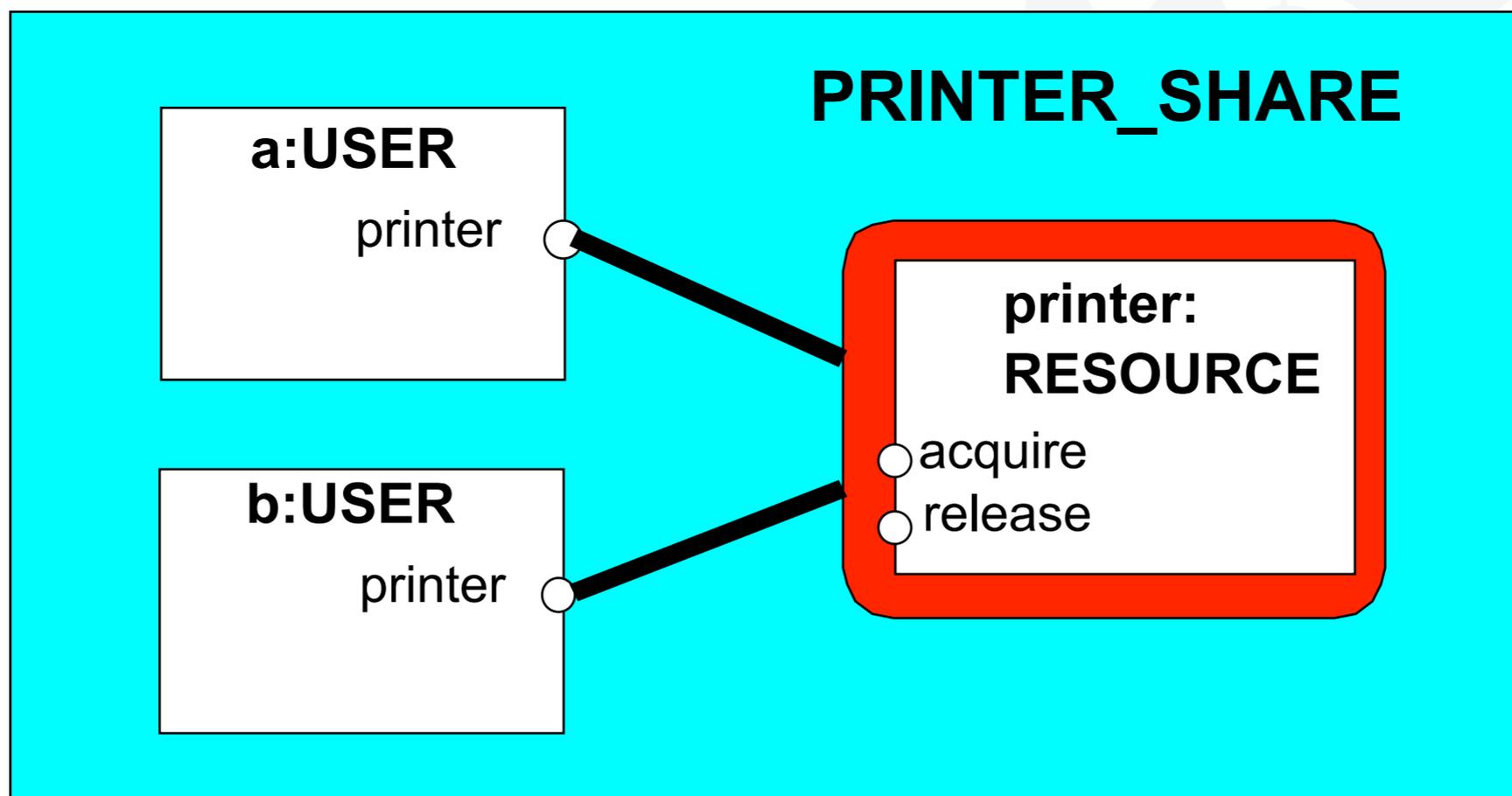


Structure Diagrams - Resource Sharing

```
RESOURCE = (acquire->release->RESOURCE) .  
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```

```
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```

Shared resources are shown as "rounded rectangles":





Java



ThreadDemo Model





ThreadDemo Model

```
THREAD = OFF,  
OFF = (toggle->ON
```



ThreadDemo Model

```
THREAD = OFF,  
OFF = (toggle->ON  
      | abort->STOP) ,  
ON   = (toggle->OFF  
      | output->ON  
      | abort->STOP) .
```



ThreadDemo Model

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THREAD = OFF ,  
  
OFF = (toggle->ON  
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ON   = (toggle->OFF  
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|| THREAD_DEMO =  
   (a:THREAD || b:THREAD)  
   /{stop/{a,b}.abort} .
```

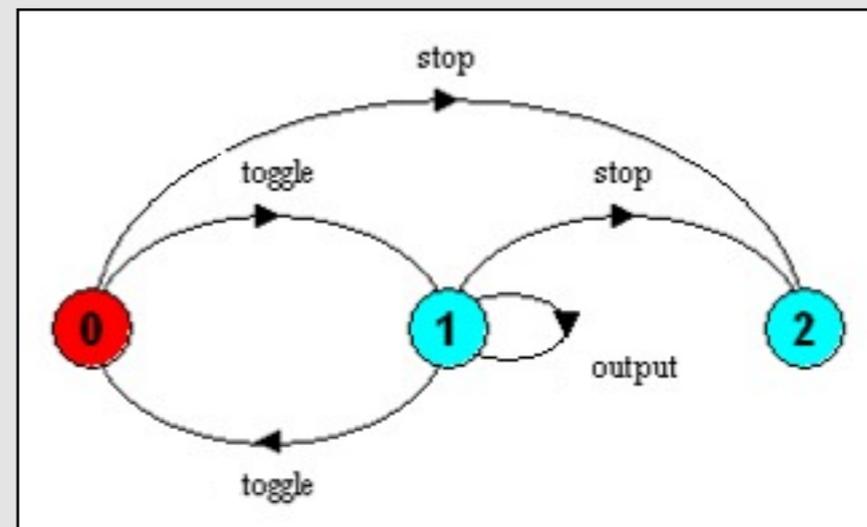
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| abort->STOP) .
```

```
|| THREAD_DEMO =  
  (a:THREAD || b:THREAD)  
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```



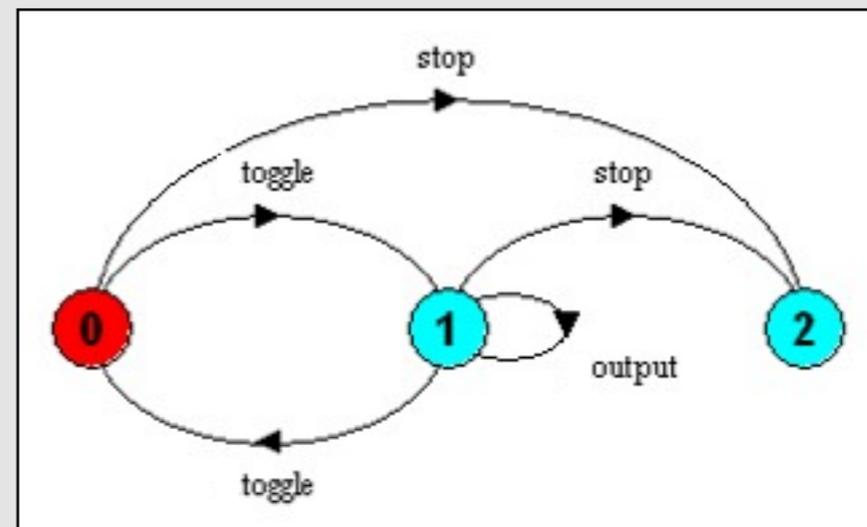
ThreadDemo Model

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THREAD = OFF,
```

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OFF = (toggle->ON  
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Interpret:

toggle, *abort*
as inputs;

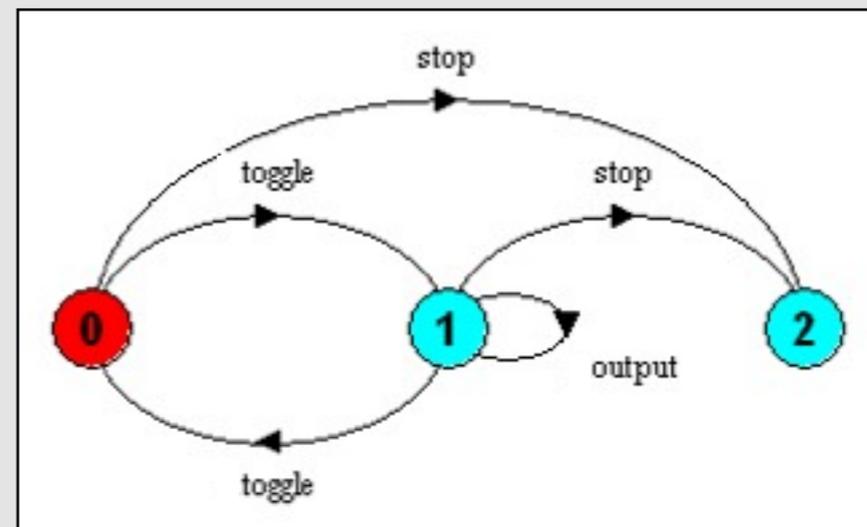
output
as output

ThreadDemo Model

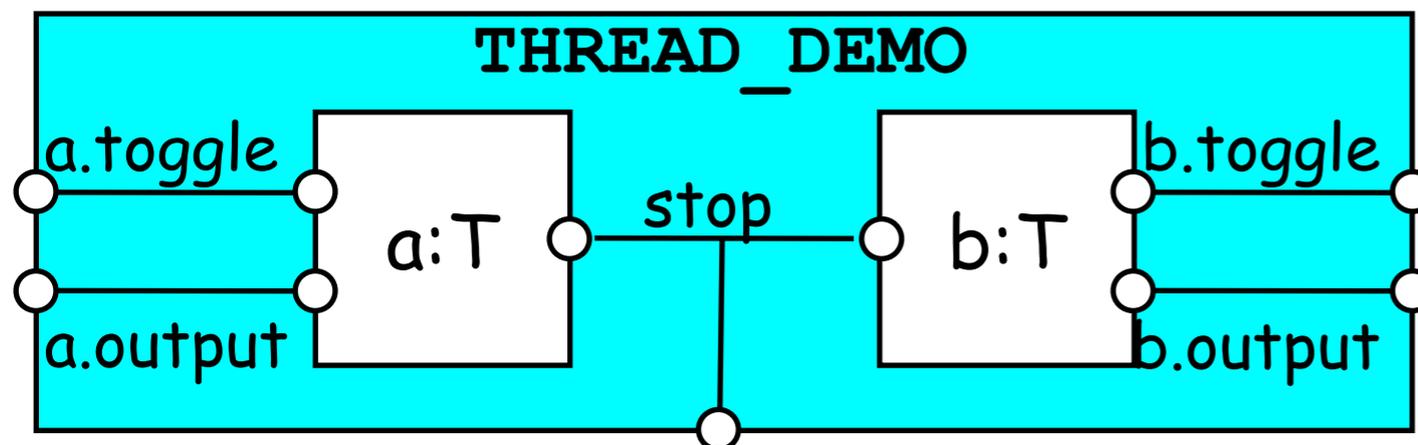
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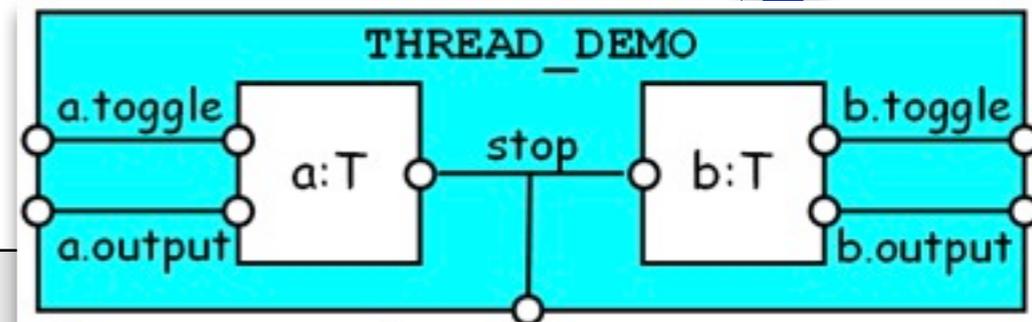
Interpret:

toggle, *abort*
as inputs;

output
as output



ThreadDemo Code: MyThread



```

class MyThread extends Thread {
    private boolean on;

    MyThread(String name) { super(name); this.on = false; }

    public void toggle() { on = !on; }

    public void abort() { this.interrupt(); }

    private void output() {
        System.out.println(getName()+" : output");
    }

    public void run() {
        try {
            while (!interrupted()) {
                if (on) output();
                sleep(500);
            }
        } catch (Int'Exc' _) {}
        System.out.println("Done!");
    }
}

```

```

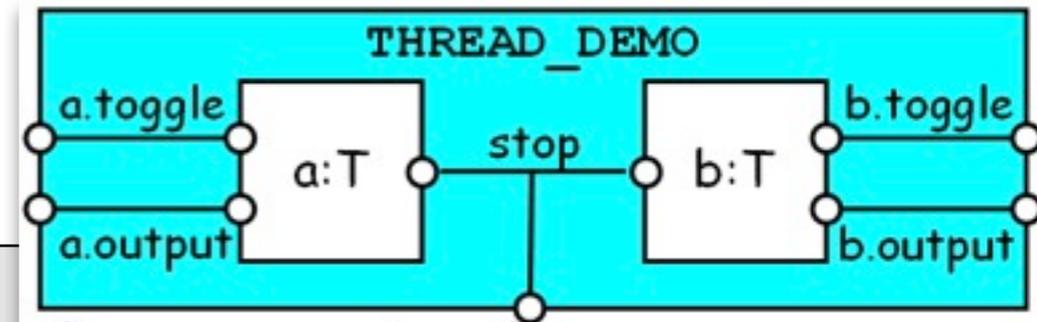
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||THREAD_DEMO =
(a:THREAD || b:THREAD)
/{stop/{a,b}.abort}.

```



ThreadDemo Code: ThreadDemo



```

class ThreadDemo {
    public static void main(String[] args) {
        MyThread a = new MyThread("a");
        MyThread b = new MyThread("b");
        a.start(); b.start();
        while (true) {
            switch (readChar()) {
                case 'a': a.toggle();
                        break;
                case 'b': b.toggle();
                        break;
                case 'i': stop(a,b);
                        return;
            }
        }
    }
    private stop(MyThread a, MyThread b) {
        a.abort();
        b.abort();
    }
}

```

```

THREAD = OFF,
OFF = (toggle->ON
      | abort->STOP),
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||THREAD_DEMO =
(a:THREAD || b:THREAD)
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```

Summary





Summary

◆ Concepts





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- Concurrent processes and process interaction





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◆ Models





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- Asynchronous (arbitrary speed) & interleaving (arbitrary order).





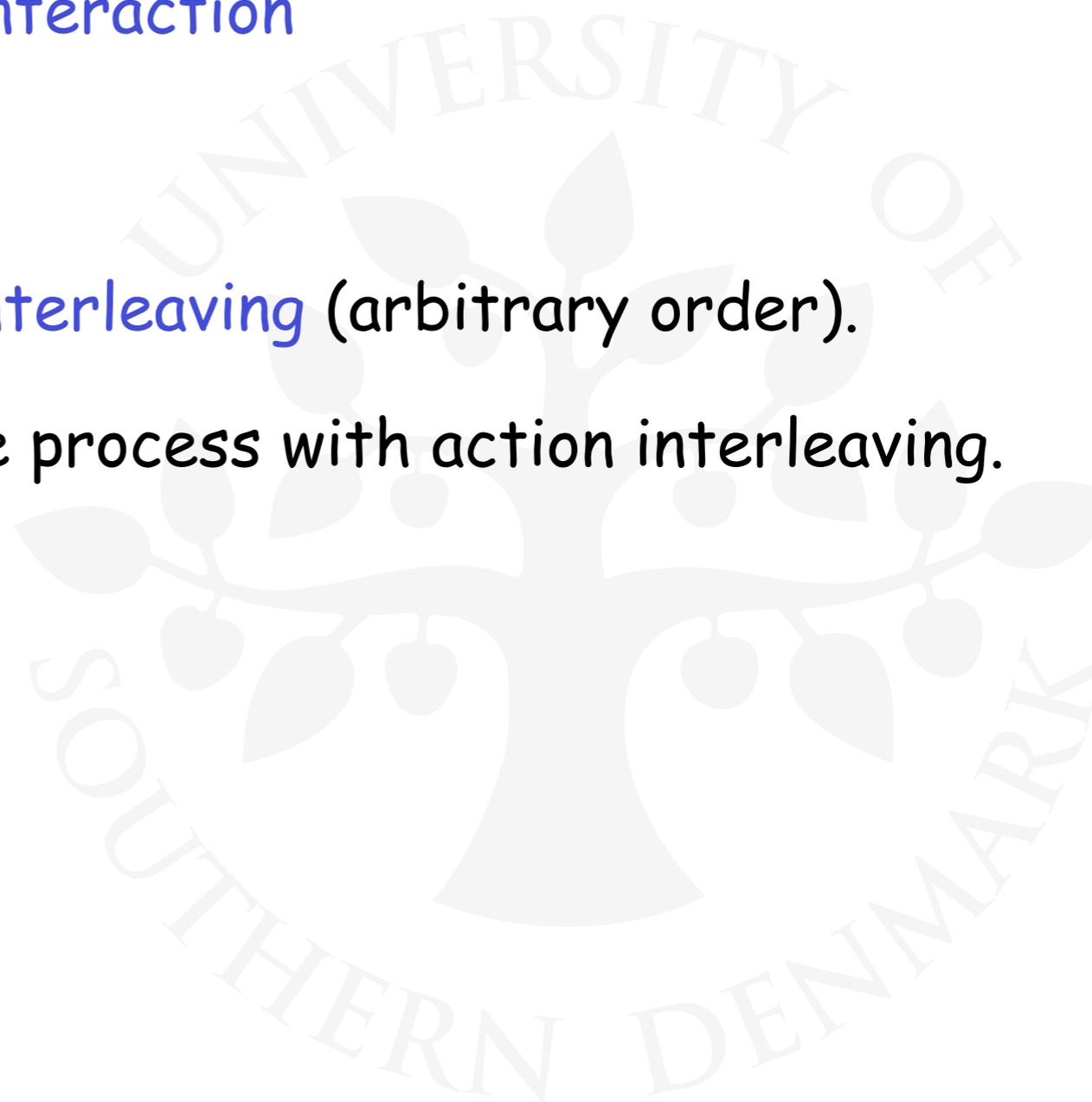
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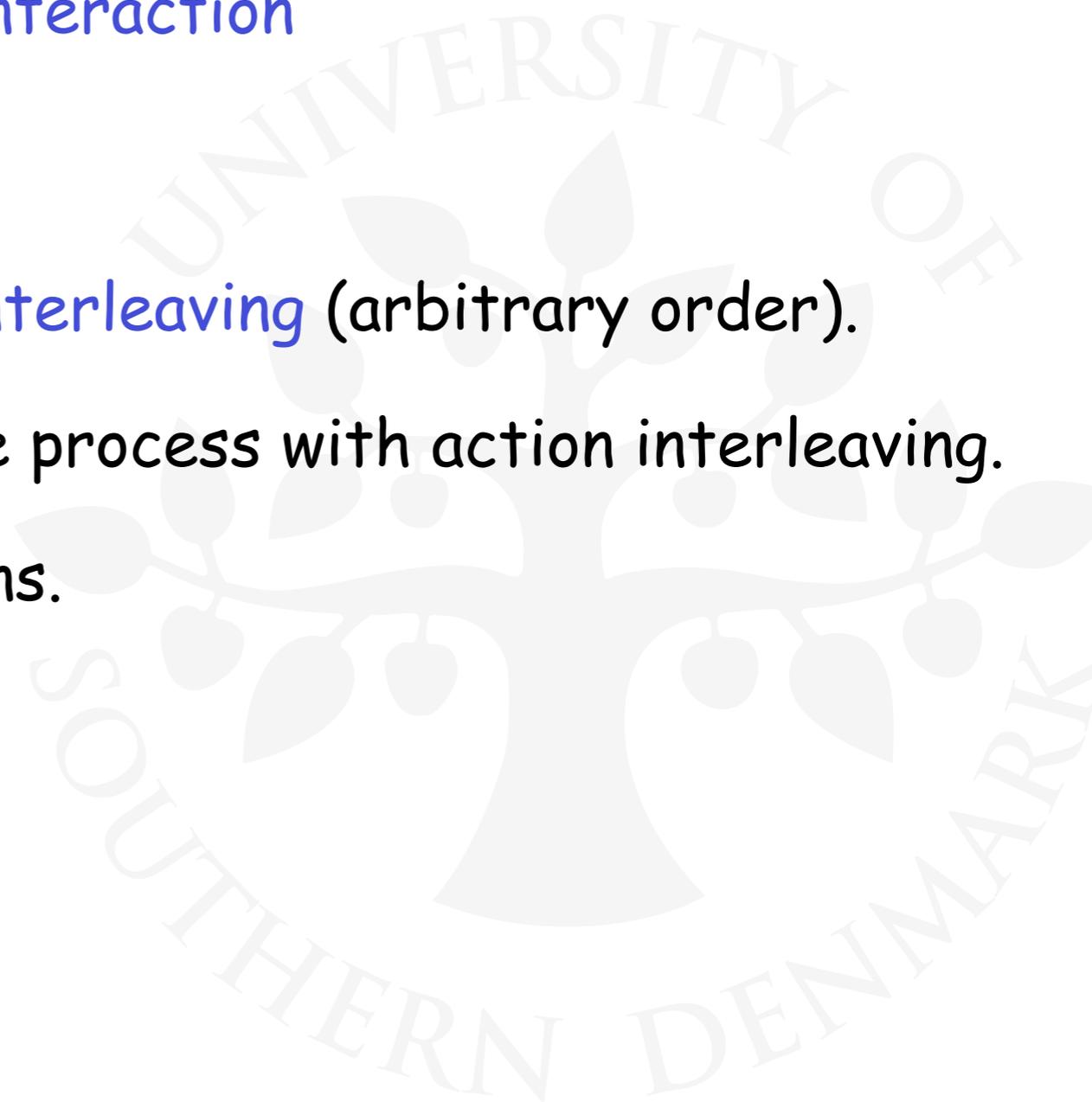
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◆ Practice

- Multiple threads in Java.