$(H, \mathcal{Q})$-COLOURINGS OF GRAPHS

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joint work with

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We consider finite undirected graphs without loops or multiple edges. Let $\mathcal{I}$ denote the class of all graphs. A *property (class) of graphs* is any nonempty class of graphs from $\mathcal{I}$ which is closed under isomorphisms. A property $\mathcal{P}$ is called (*induced*) *hereditary* if every (induced) subgraph of any graph with property $\mathcal{P}$ also has property $\mathcal{P}$.

Let be given graphs G and H, a *homomorphism* of G to H is a vertex-mapping $h : V(G) \to V(H)$ such that $vv' \in E(G)$ implies $h(v)h(v') \in E(H)$. If there exists a homomorphism of G to H, then we write $G \to H$. Clearly the existence of a homomorphism $G \to K_k$ coincides with the existence of a k-colouring of G and thus homomorphisms $G \to H$ are called H-*colourings*, where labels of vertices from $V(H)$ are referred to as colours.

Let $\mathcal{Q} \subseteq \mathcal{B}$, where $\mathcal{B}$ denotes the class of bipartite graphs, and let G be H-coloured such that for any two colour classes V_i, V_j the graph $G[V_i \cup V_j] \in \mathcal{Q}$. Such homomorphism will be denoted by $G \stackrel{\mathcal{Q}}{\to} H$ and called $(H, \mathcal{Q})$-*colouring.*

Let $\mathcal{P}$ be a hereditary property of graphs and assume that $k \geq 2$.
We say that $\mathcal{P}$ is a $(k, \mathcal{Q})$-*Ramsey class*, if for every $G \in \mathcal{P}$ there exists $H \in \mathcal{P}$ such that for every $(K_k^*, \mathcal{Q})$-colouring of H (K_k^* denotes a k-complete reflexive graph) there is a monochromatic copy of G.

In a talk some $(k, \mathcal{Q})$-Ramsey classes and results concerning $(H, \mathcal{Q})$-colourings, for selected H and $\mathcal{Q}$, will be presented.