Minimum Degree and Density of Binary Sequences

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For $d, k \in \mathbb{N}$ with $k \leq 2d$, let $g(d,k)$ denote the infimum density of binary sequences $(x_i)_{i \in \mathbb{Z}} \in \{0,1\}^{\mathbb{Z}}$ which satisfy the minimum degree condition $\sum_{j=1}^{d}(x_{i+j} + x_{i-j}) \geq k$ for all $i \in \mathbb{Z}$ with $x_i = 1$. For $k \leq d$ we get $g(d,k) = 0$. For $d+1 \leq k \leq 2d$ we reduce the calculation of $g(d,k)$ to the case of cyclic sequences of bounded length. This is related to the generalized girth of powers of cycles, investigated earlier by Kézdy and Markert, and by Bermond and Peyrat. For a graph G the generalized k-girth is defined as the minimum order of an induced subgraph of minimum degree at least k. We present a minimum mean cycle formulation which allows to compute $g(d,k)$ exactly for small values of d and k. For even values of $k > d$ we have $g(d,k) = \frac{k}{2d}$ (Bermond and Peyrat). For odd $k > d$ we show that $g(d,k) \leq \frac{k^2-1}{2(dk-1)}$. Moreover, we conjecture that equality holds, as well as the structure of the optimal sequences attaining this bound. We prove the conjecture for $k \geq 2d-3$.
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