Three complexity results on coloring P_k-free graphs

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Abstract

We prove three complexity results on vertex coloring problems restricted to P_k-free graphs, i.e., graphs that do not contain a path on k vertices as an induced subgraph. First of all, we show that the pre-coloring extension version of 5-coloring remains NP-complete when restricted to P_6-free graphs. Recent results of Hoàng et al. imply that this problem is polynomially solvable on P_5-free graphs. Secondly, we show that the pre-coloring extension version of 3-coloring is polynomially solvable for P_6-free graphs. This implies a simpler algorithm for checking the 3-colorability of P_6-free graphs than the algorithm given by Randerath and Schiermeyer. Finally, we prove that 6-coloring is NP-complete for P_7-free graphs. This problem was known to be polynomially solvable for P_5-free graphs and NP-complete for P_8-free graphs, so there remains one open case.