How to generalize Gallai colorings?

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Gallai-colorings (G-colorings)of complete graphs are edge colorings such that there is no rainbow triangle, a triangle with three distinct colors. G-colorings occur in various contexts such as the theory of partially ordered sets (in Gallai's original paper), information theory and the theory of perfect graphs. There are several interesting properties of G-colorings, for example

- Disconnection property: G-colorings with at least three colors have a disconnected color class

- Connection property: G-colorings have a connected color class spanning all vertices

- Substitution property: any G-coloring can be obtained by substituting G-colored complete graphs into a nontrivial 2-colored complete graph.

The talk gives initial results and problems about possible generalizations of G-colorings, such as

- G-coloring of non-complete graphs - keeping the same definition, no rainbow triangles.

  Appetizer: Suppose G is a graph containing no three pairwise nonadjacent vertices. Then G has a monochromatic connected subgraph with at least $\frac{|V(G)|}{5}$ vertices in every G-coloring. Can one improve on this?

- forbidding a rainbow H instead of rainbow triangles

- coloring hypergraphs - triples without rainbow tetrahedron