Minimum Makespan Multi-vehicle Dial-a-Ride

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Abstract

Dial-a-Ride problems consist of a set V of n vertices in a metric space (denoting travel time between vertices) and a set of m objects represented as source-destination pairs $\{(s_i, t_i)\}_{i=1}^m$, where each object requires to be moved from its source to destination vertex. In the *multi-vehicle Dial-a-Ride* problem, there are q vehicles each having capacity k and where each vehicle $j \in [q]$ has its own depot-vertex $r_j \in V$. A feasible schedule consists of a capacitated route for each vehicle (where vehicle j originates and ends at its depot r_j) that together move all objects from their sources to destinations. The objective is to find a feasible schedule that minimizes the maximum completion time (i.e. *makespan*) of vehicles, where the completion time of vehicle j is the time when it returns to its depot r_j at the end of its route. We consider the *preemptive* version of multi-vehicle Dial-a-Ride, where an object may be left at intermediate vertices and transported by more than one vehicle, while being moved from source to destination. Approximation algorithms for the single vehicle Dial-a-Ride problem ($q = 1$) have been considered in [Charikar and Raghavachari FOCS'98, Gupta et al. ESA'07].

Our main results are an $O(\log^3 n)$-approximation algorithm for *preemptive multi-vehicle Dial-a-Ride*, and an improved $O(\log t)$-approximation for its special case when there is no capacity constraint (here $t \leq n$ is the number of distinct depot-vertices). Our algorithm is based on an interesting recursive framework, and uses a new structural property of single-vehicle Dial-a-Ride tours. The improvement in the uncapacitated case comes from the use of sparse spanners in constructing the routes traced by objects. There is an $\Omega(\log^{1/4} n)$ hardness of approximation known [Gørtz APPROX'06] even for single vehicle capacitated preemptive Dial-a-Ride. We also obtain an improved constant factor approximation algorithm for the uncapacitated multi-vehicle problem on metrics induced by graphs excluding any fixed minor.

Our results represent the first work to address the multi-vehicle preemptive problem which have traditionally been ignored not only in the approximation algorithms literature but also in the design of exact methods, owing perhaps to the difficulty of representing the combinatorial explosion of transshipment vertices as well as the movement of objects across various vehicles. Our methods address these difficulties using new lower bounds as well as new algorithmic structures such as spanners, load balancing via matchings and recursion.

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