# Body-bar graphs with unique d-dimensional realizations

Tibor Jordán

Department of Operations Research, Eötvös University, Budapest

## abstract

A basic geometric question is whether a given geometric graph (also called *framework*) $(G, p)$ is uniquely realized in $\mathbb{R}^d$. In this notation $G$ is a graph and $p$ is a configuration of points corresponding to the vertices of $G$. The framework is *uniquely realized* if for any other configuration $q$ for $G$ such that the edge lengths of $(G, q)$ are the same as the corresponding edge lengths of $(G, p)$, we have that $q$ is congruent to $p$.

Recent results have shown that when the configuration $p$ is *generic* and $d = 2$, there is a good combinatorial characterization of when $(G, p)$ is uniquely realized in $\mathbb{R}^2$ that only depends on $G$. It appears that a similar result for $d = 3$ is beyond the scope of present techniques. However, there is a special class of frameworks that are more amenable and which can be applied in the analysis of the flexibility of molecules. These are constructed from a finite number of 'rigid bodies' that are connected by bars generically placed with respect to each body. We call the graph of such a framework a *body-bar* graph.

By using techniques from combinatorial rigidity and graph theory we characterize the body-bar graphs $G$ for which a generic framework $(G, p)$ is uniquely realized in $\mathbb{R}^d$, for any $d \geq 1$. As a consequence there is a deterministic polynomial time algorithm to determine the unique realizability of generic body-and-bar frameworks in any dimension.

This is joint work with Robert Connelly (Cornell) and Walter Whiteley (Toronto).