Generalized Colouring and Fractional Invariants of Graphs

PETER MIHÓK
(Faculty of Economics TU and Mathematical Institute of SAS, Košice, Slovakia)

A graph property $\mathcal{P}$ is any nonempty isomorphism-closed class of simple (finite or infinite) graphs. We will consider additive and hereditary graph properties i.e. classes closed under disjoint union and subgraphs.
Let $\omega(G); \chi_f(G); \chi_c(G); \chi(G); ch(G); col(G); \Delta(G)$ be the clique number; fractional chromatic number; circular chromatic number; chromatic number, choice number, colouring number and maximum degree of a graph G, respectively .It is well known, that for any graph G we have

$$\omega(G) \leq \chi_f(G) \leq \chi_c(G) \leq \chi(G) \leq ch(G) \leq col(G) \leq \Delta(G) + 1.$$

In our talk we will introduce the generalized versions of above mentioned chain of invariants and present some basic results on this topic.

In 1999 (see [1]) we considered with Zsolt Tuza and Margit Voigt the generalized fractional chromatic number of graph. This concept leads to the following notion. Let a, b be positive integers, $a > b$ and $\mathcal{P}_1, \mathcal{P}_2, \dots \mathcal{P}_a$ be additive and hereditary graph properties. A *fractional* $(\mathcal{P}_1, \mathcal{P}_2, \dots \mathcal{P}_a; b)$-colouring of a graph G is a mapping ϕ of $V(G)$ to the set of b-element subsets of $\{1, 2, \dots, a\}$ such that for each "'colour"' $i, 1 \leq i \leq a$ the subgraph $G[i]$ induced by the vertices where $i \in \phi(v)$ has the property $\mathcal{P}_i$. We will describe the structure of the classes of fractionally $(\mathcal{P}_1, \mathcal{P}_2, \dots \mathcal{P}_a; b)$-colourable graphs and we will show e.g. that planar graphs are fractionally $(\mathcal{P}_1, \mathcal{P}_2, \dots \mathcal{P}_5; 2)$-colourable for $\mathcal{P}_i$ "'to be acyclic"'.