Chromatic number and complete graph substructures for degree sequences

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November 1, 2009

(Abstract:) Given a graphic degree sequence D, let $\chi(D)$ (respectively $\omega(D)$, $h(D)$, and $H(D)$) denote the maximum value of the chromatic number (respectively, the size of the largest clique, largest clique subdivision, and largest clique minor) taken over all graphs whose degree sequence is D. It is proved that $\chi(D) \leq h(D)$. Moreover, it is shown that a subdivision of a clique of order $\chi(D)$ exists where each edge is subdivided at most once and the set of all subdivided edges forms a collection of disjoint stars. This bound is an analogue of the Hajós Conjecture for degree sequences and, in particular, settles a conjecture of Neil Robertson that degree sequences satisfy the bound $\chi(D) \leq H(D)$ (which is related to the Hadwiger Conjecture). It is also proved that $\chi(D) \leq \frac{6}{5}\omega(D) + \frac{3}{5}$ and that $\chi(D) \leq \frac{4}{5}\omega(D) + \frac{1}{5}\Delta(D) + 1$, where $\Delta(D)$ denotes the maximum degree in D. The latter inequality is a strengthened version of a conjecture of Bruce Reed. All derived inequalities are best possible. This is a joint work with Zdenek Dvorak.