Bounds on the Independence Number of a Graph

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We discuss three recent results concerning independence in graphs.

- For a non-negative integer T, we prove that the independence number of a graph G in which every vertex belongs to at most T triangles is at least $\sum\limits_{u \in V} f(d(u), T)$ where $d(u)$ denotes the degree of a vertex $u \in V$, $f(d,T) = \frac{1}{d+1}$ for $T \geq \binom{d}{2}$ and $f(d,T) = (1 + (d^2 - d - 2T)f(d-1,T))/(d^2 + 1 - 2T)$ for $T < \binom{d}{2}$. This is a common generalization of the lower bounds for the independence number due to Caro, Wei, and Shearer. We discuss further possible strengthenings of our result and pose a corresponding conjecture.

  (*Joint work with A. Boßecker.*)

- We prove that if G is a graph with κ components and independence number $\alpha(G)$, then there exist a positive integer $k \in \mathbb{N}$ and a function $f : V \to \mathbb{N}_0$ with non-negative integer values such that $f(u) \leq d(u)$ for $u \in V$, $\alpha(G) \geq k \geq \sum\limits_{u \in V} \frac{1}{d(u)+1-f(u)}$, and $\sum\limits_{u \in V} f(u) \geq 2(k - \kappa)$. This result is a best-possible improvement of a result due to Harant and Schiermeyer [On the independence number of a graph in terms of order and size, *Discrete Math.* **232** (2001), 131-138] and implies that $\frac{\alpha(G)}{n(G)} \geq \frac{2}{\left(d(G)+1+\frac{2}{n(G)}\right)+\sqrt{\left(d(G)+1+\frac{2}{n(G)}\right)^2-8}}$ for connected graphs G of order $n(G)$ and average degree $d(G)$.

  (*Joint work with J. Harant.*)

- We prove several best-possible lower bounds in terms of the order and the average degree for the independence number of graphs which are connected and/or satisfy some odd girth condition. Our main result is the extension of a lower bound for the independence number of triangle-free graphs of maximum degree at most 3 due to Heckman and Thomas [A New Proof of the Independence Ratio of Triangle-Free Cubic Graphs, *Discrete Math.* **233** (2001), 233-237] to arbitrary triangle-free graphs. For connected triangle-free graphs of order n and size m, our result implies the existence of an independent set of order at least $(4n - m - 1)/7$.

  (*Joint with C. Löwenstein, A.S. Pedersen, and F. Regen.*)