The Erdős-Lovász Tihany Problem

Michael Stiebitz

Technische Universität Ilmenau, Germany

At the Graph Theory Conference held in Tihany, Hungary, 1966, Paul Erdős asked whether every 5-chromatic graph not containing a K_5 has two vertex disjoint odd cycles. Lovász generalized this question as follows:

The Erdős-Lovász Tihany Problem 1966: *Let G be a graph and let $2 \leq s \leq t$ be natural numbers such that $\chi(G) = s + t - 1$ and $\chi(G) > \omega(G)$. Does there exists a pair of vertex disjoint subgraphs of G of chromatic number s and t, respectively?*

For the special case $s = 2$, the problem can be formulated as follows: A k-chromatic graph G is said to be **double-critical** if $G - x - y$ is $(k - 2)$-colourable whenever $xy \in E(G)$. Is K_k the only double-critical k-chromatic connected graph?

The question of Erdős ($s = t = 3$) was settled in the affirmative by Brown and Jung in 1969. Up to now, the problem is solved in the affirmative only if $(s, t) \in \{(2, 2), (2, 3), (2, 4), (3, 3), (3, 4), (3, 5)\}$. In the talk we discuss some recent results related to the Erdős-Lovász Tihany Problem.
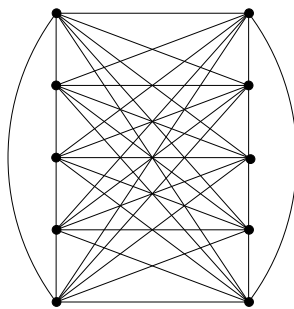

Figure 1: A 6-chromatic graph with many double-critical edges