Covering line graphs with equivalence relations

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Abstract

An equivalence graph is a disjoint union of cliques, and the equivalence number $eq(G)$ of a graph G is the minimum number of equivalence subgraphs needed to cover the edges of G. We consider the equivalence number of a line graph, giving improved upper and lower bounds: $\frac{1}{3}\log_2\log_2\chi(G) < eq(L(G)) \leq 2\log_2\log_2\chi(G) + 2$, where $\chi(G)$ stands for the chromatic number fo G. This disproves a recent conjecture that $eq(L(G))$ is at most three for triangle-free G; indeed it can be arbitrarily large.

To bound $eq(L(G))$ we bound the closely-related invariant $\sigma(G)$, which is the minimum number of orientations of G such that for any two edges e, f incident to some vertex v, both e and f are oriented out of v in some orientation. When G is triangle-free, $\sigma(G) = eq(L(G))$. We also prove that even when G is triangle-free, it is NP-complete to decide whether or not $\sigma(G) \leq 3$.

References

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