A complexity dichotomy for the coloring of sparse graphs*

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Abstract

Gallucio, Goddyn and Hell proved in 2001 that in any minor-closed class, graphs with large enough girth have a homomorphism to any given odd cycle. In this paper, we study the computational aspects of this problem. We show that for any minor-closed class $\mathcal{F}$ containing all planar graphs, and such that all minimal obstructions are 3-connected, the following holds: for any k there is a $g = g(\mathcal{F}, k)$ such that every graph of girth at least g in $\mathcal{F}$ has a homomorphism to C_{2k+1}, but deciding whether a graph of girth $g - 1$ in $\mathcal{F}$ has a homomorphism to C_{2k+1} is NP-complete. The classes of graphs on which this result applies include planar graphs, K_n-minor free graphs, and graphs with bounded Colin de Verdière parameter (for instance, linklessly embeddable graphs).

We also show that the same dichotomy occurs in problems related to a question of Havel (1969) and a conjecture of Steinberg (1976) about the 3-colorability of sparse planar graphs.

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