Structural description of dense graphs with large odd girth

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A graph has odd girth at least $2k+1$ if it does not contain an odd cycle of length less than $2k+1$. Brandt and Ribe-Baumann verified a conjecture of Albertson, Chan, Haas from 1993, by showing that every graph of order n with odd girth at least $2k+1$ and minimum degree $\delta \geq 3n/4k$ is either homomorphic with C_{2k+1} or it is a regular blown up Möbius ladder with $2k$ rungs. Here we analyse the structure of the graphs if the minimum degree is allowed to be somewhat smaller. We formulate a general framework of statements in the above sense. For the triangle-free case $k=2$, we give a slight refinement of a sequence of theorems by Jin 1993 with a very short proof. For the case $k=3$ we prove the next statement for minimum degree $\delta \geq 4n/17$, and for the general case we will present structural refinements and properties of the limiting graphs.

Partly joint work with Elizabeth Ribe-Baumann (Ilmenau).