Separator Theorems for Minor-Free and Shallow Minor-Free Graphs with Applications

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Abstract

Alon, Seymour, and Thomas generalized Lipton and Tarjan's planar separator theorem and showed that for a graph H with h vertices, an H-minor free graph with n vertices has a separator of size at most $h^{3/2}\sqrt{n}$. They gave an algorithm that, given a graph G with m edges and n vertices, outputs in $O(\sqrt{hn}m)$ time such a separator or a minor of G isomorphic to H. Plotkin, Rao, and Smith gave an $O(hm\sqrt{n\log n})$ time algorithm to find a separator of size $O(h\sqrt{n\log n})$. Kawarabayashi and Reed improved the bound on the size of the separator to $h\sqrt{n}$ and gave an algorithm that finds such a separator in $O(n^{1+\epsilon})$ time for any constant $\epsilon > 0$, assuming h is constant. This algorithm has an extremely large dependency on h in the running time (some power tower of h whose height is itself a function of h), making it impractical even for small h. We are interested in a small polynomial time dependency on h and we show how to find an $O(\text{poly}(h)\sqrt{n\log n})$-size separator or report that a minor of G isomorphic to H exists in $O(\text{poly}(h)n^{5/4+\epsilon})$ time for any constant $\epsilon > 0$. We also present the first $O(\text{poly}(h)n)$ time algorithm to find a separator of size $O(n^c)$ for a constant $c < 1$. As corollaries of our results, we get improved algorithms for shortest paths and maximum matching. Furthermore, for integers l and h, we give an $O(m + n^{2+\epsilon}/l)$ time algorithm that either produces a K_h-minor of depth at most $l\log n$ or a separator of size at most $O(n/l + lh^2\log n)$. This improves the shallow minor algorithm of Plotkin, Rao, and Smith when $m = \Omega(n^{1+\epsilon})$. We get a similar improvement for constructing cut covers of a graph.