Graph Minor Algorithm with the parity condition*

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Abstract

We generalize the seminal Graph Minor algorithm to the parity version. Our polynomial time algorithms include the following problems:

1. The parity H-minor (Odd K_k-minor) containment problem.
2. The disjoint paths problem with k terminals and the parity condition for each path.

We present an $O(m\alpha(m,n)n)$ time algorithm for these problems for any fixed k, where n, m are the number of vertices and the number of edges, respectively, and the function $\alpha(m,n)$ is the inverse of the Ackermann function.

Note that the first problem includes the problem of testing whether or not a given graph contains k disjoint odd cycles (which was recently solved by two of the authors), if H consists of k disjoint triangles. Our algorithm for the second problem clearly generalizes the Graph Minor algorithm by Robertson and Seymour for the disjoint paths problem with k terminals, because we can test every possible parity conditions for each path (there are 2^k possibilities), which would give rise to the solution for the (nonparity) disjoint paths problem. Indeed, we generalize all the results in the Graph Minor algorithm to the parity version.

As with the Robertson-Seymour algorithm to solve the k disjoint paths problem for any fixed k, in each iteration, we would like to either use a huge clique minor as a "crossbar", or exploit the structure of graphs in which we cannot find such a minor. Here, however, we must take care of the parity of the paths and can only use an "odd clique minor". We must also describe the structure of those graphs in which we cannot find such a minor and discuss how to exploit it.

Let us emphasis that our proof for the correctness of the above algorithms does not depend on the full power of the Graph Minor structure theorem. Although the Graph Minor algorithm does depend on it and our proof has some similarity, we can avoid the structure theorem, because there is now a shorter proof for the correctness of the graph minor algorithm by two of the authors. Utilizing some results there, we are able to avoid the much of the heavy machinery of the Graph Minor structure theory, and give a proof within 30 pages.

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