

2-Colorings in k -Regular k -Uniform Hypergraphs and Cycles in Digraphs

Michael A. Henning* and Anders Yeo†

Abstract

A k -regular k -uniform hypergraph is a hypergraph where every edge contains k vertices and every vertex belongs to k -edges. A hypergraph is 2-colorable if there is a 2-coloring of the vertices with no monochromatic hyperedge. We will discuss the result that if $k \geq 4$ then every k -regular k -uniform hypergraph is 2-colorable. This result is known not to hold for $k = 3$.

We will then discuss the conjecture that there always exists a set of $k - 3$ vertices in a k -regular k -uniform hypergraph which can be left uncolored by a 2-coloring such that all edges still contain vertices of both colors.

We will discuss generalisations of the above conjecture as well as some application. In particular we will show how the above results (and related results) are connected to the existence of even cycles in regular digraphs and disjoint cycles in digraphs. We will state the new result that all 4-regular digraphs contain vertex disjoint cycles of different length and conjecture that this is also true for 3-regular bipartite digraphs.

*Department of Mathematics, University of Johannesburg, Auckland Park, 2006 South Africa.

†Department of Computer Science, Royal Holloway, University of London, Egham, Surrey TW20 OEX, UK.