The circumference of the square of a connected graph

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Fleischners celebrated result states that the square of every two-connected graph is hamiltonian. We investigate how small the circumference of the square of a connected graph of order n can be. We present a lower bound for the smallest circumference which is logarithmic in n. This bound is tight for infinitely many values of n and for each such n there is a unique tree of order n attaining the bound.
Harary and Schwenk proved that the square of a tree of order $n \geq 3$ is hamiltonian, if and only if the tree is a caterpillar, i.e. a tree where the deletion of all its leaves leaves a path. Let $S_{r,r,r}$ denote the graph obtained from a claw $K_{1,3}$ by subdividing each edge exactly $r-1$ times. It is easy to see that a tree is a caterpillar if and only if it does not contain $S_{2,2,2}$ as a subgraph. We investigate the question how small the circumference of the square of a tree that does not contain $S_{r,r,r}$ as a subgraph can be. For the case $r = 3$ we present a lower bound for the circumference of the square which is of the order of the square root of n and which is again tight for infinitely many values of n.