Graph Powers and Graph Colorings

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(Joint work with **Ali Taherkhani**)

Abstract

In this talk, we introduce two kinds of power for graphs [1, 2]. First, for a given graph G, we consider $G^{\frac{r}{s}}$, i.e., the rth power of the sth subdivision of G, and we present some basic properties of this power. In the sequel, we introduce the graph power $G^{\widetilde{\frac{2s+1}{2r+1}}}$. We show that these powers can be considered as the dual of each other. Precisely, we show that if $\frac{2r+1}{2s+1} < og(G)$ and $2s+1 < og(H^{\widetilde{\frac{1}{2r+1}}})$, then

$$G^{\frac{2r+1}{2s+1}} \longrightarrow H \Longleftrightarrow G \longrightarrow H^{\widetilde{\frac{2s+1}{2r+1}}}.$$

Next, we review some coloring properties of graph powers [3]. We show that if $\frac{2r+1}{2s+1} \leq \frac{\chi_c(G)}{3(\chi_c(G)-2)}$, then $\chi_c(G^{\frac{2r+1}{2s+1}}) = \frac{(2s+1)\chi_c(G)}{(s-r)\chi_c(G)+2r+1}$. In particular, one can see that $\chi_c(K_{3n+1}^{\frac{1}{3}}) = \frac{9n+3}{3n+2}$ and $K_{3n+1}^{\frac{1}{3}}$ has no subgraph with circular chromatic number equal to $\frac{6n+1}{2n+1}$. This provides a negative answer to a question asked in [Xuding Zhu, Circular chromatic number: a survey, Discrete Math., 229(1-3):371–410, 2001]. Also, we present an upper bound for the fractional chromatic number of subdivision graphs. Precisely, we show that $\chi_f(G^{\frac{1}{2s+1}}) \leq \frac{(2s+1)\chi_f(G)}{s\chi_f(G)+1}$.

Keywords: graph homomorphism, circular coloring, fractional chromatic number.
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