Critical graphs and hypergraphs with few edges

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A graph G is called *k-critical* if $\chi(H) < \chi(G) = k$ for every proper subgraph H of G. Clearly, K_k is a k-critical graph and for $k = 1, 2$ there are no other k-critical graphs. König's theorem implies that the only 3-critical graphs are the odd cycles. However, for a given integer $k \geq 4$, a characterization of all k-critical graphs seems unattainable. Critical graphs were first defined and investigated by Dirac in the 1950s. In particular, Dirac investigated the function

$$f_k(n) = \min\{|E(G)| \mid G \text{ is } k\text{-critical and} |V(G)| = n\}.$$

If $k \geq 4$, this function is defined for all n with $n \geq k$ and $n \neq k+1$. Since any k-critical graph has minimum degree at least $k-1$, we have $f_k(n) \geq \frac{1}{2}(k-1)n$. Brooks' theorem implies that $f_k(n) = \frac{1}{2}(k-1)n$ if and only if $n = k$. Recently, Yancey and Kostochka found the best linear approximation for the function $f_k(n)$. We also consider the corresponding function for k-critical hypergraphs and k-list-critical graphs.

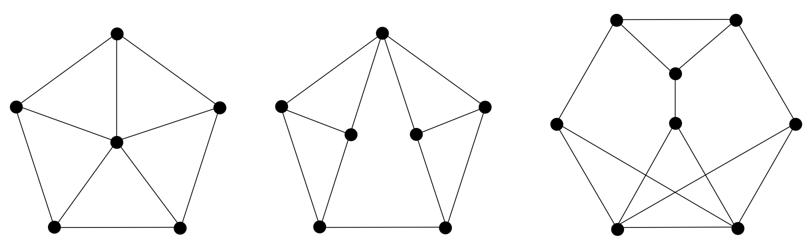

Figure 1: The only 4-critical graphs of order $n = 6, 7, 8$ with $f_4(n)$ edges.