Robustly expanding directed graphs

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Roughly speaking, a directed graph G is a robust outexpander if for every set S of vertices which is not too small and not too large, the 'robust outneighbourhood' of S is at least a little larger than S, i.e. there are slightly more than $|S|$ vertices having many inneighbours in S. This notion has turned out to be extremely useful: Every robust outexpander G, whose minimum in- and outdegrees are linear in the number of vertices of G, has a Hamilton cycle. Moreover, if in addition G is regular, then it even has a Hamilton decomposition.

Many important classes of directed graphs are robust outexpanders, for example regular tournaments, random tournaments as well as directed graphs satisfying certain degree conditions. So results about robust outexpanders have applications to several classes of (directed) graphs. For example, the above result about Hamilton decompositions in regular robust outexpanders immediately implies a conjecture of Kelly from 1968 that every regular tournament has a decomposition into edge-disjoint Hamilton cycles.

In my talk I will discuss this concept as well as some of the applications. The results are joint work with Daniela Kühn, John Lapinskas, Katherine Staden and Andrew Treglown.