

Fractional Colouring and Pre-colouring Extension of Graphs

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Suppose we are given a graph in which some vertices are already pre-coloured, and we want to extend this partial colouring to a colouring of the whole graph. Because of the pre-coloured vertices, we may need more colours than just the chromatic number. How many extra colours are needed under what conditions has been well-studied.

We consider the same problem in the context of fractional colourings. One way to define fractional colourings is as follows. We are given an interval $[0, K]$ of real numbers, and we need to assign to each vertex of subset of $[0, K]$ of measure one so that adjacent vertices receive disjoint subsets. The fractional chromatic number is the minimum K for which this is possible.

Again assume that certain vertices are already pre-coloured (are already assigned a subset of measure one). If we further assume some knowledge about the pre-coloured vertices (say they are far apart), what value of K is required to guarantee that we can always extend this partial colouring to a fractional colouring of the whole graph? The answer to this questions shows a surprising dependence on the fractional chromatic number of the graph under consideration.

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