FACIT-LISTE MAT. 1+2 JUNI 2009

OPG. 1 a)

$$P_1(x) = f(a) + f'(a)\cdot(x-a)$$
$$= f(2) + f'(2)\cdot(x-2)$$
$$= 4.5 + 3(x-2) = \underline{\underline{3x-1.5}}$$

$$f(2.02) \approx P_1(2.02) = 4.5 + 3\cdot 0.02 = \underline{\underline{4.56}}$$

OPG. 1 b)

$$f(x) = x^2 + (x^2-2)^{-1}$$
$$f'(x) = 2x + (-1)\cdot(x^2-2)^{-2}\cdot 2x$$
$$f''(x) = 2 + (-1)(-2)(x^2-2)^{-3}\cdot 2x\cdot 2x + (-1)(x^2-2)^{-2}\cdot 2$$
$$= 2 + 8\cdot(x^2-2)^{-3}\cdot x^2 - 2(x^2-2)^{-2}$$
$$f''(2) = 2 + 8\cdot\tfrac{1}{8}\cdot 4 - 2\cdot\tfrac{1}{4}$$
$$= 2 + 4 - \tfrac{1}{2} = \tfrac{11}{2} = 5.5$$

$$P_2(x) = f(a) + f'(a)\cdot(x-a) + \tfrac{1}{2}\cdot f''(a)\cdot(x-a)^2$$
$$= f(2) + f'(2)\cdot(x-2) + \tfrac{1}{2}\cdot f''(2)\cdot(x-2)^2$$
$$= \underline{\underline{4.5 + 3(x-2) + \tfrac{1}{2}\cdot 5.5\,(x-2)^2}}$$
$$= 4.5 + 3(x-2) + 2.75(x^2+4-4x)$$
$$= \underline{\underline{2.75x^2 - 8x + 9.5}}$$

$$f(2.02) \approx P_2(2.02) = 4.5 + 3\cdot 0.02 + 2.75\cdot 0.02^2$$
$$= 4.5 + 0.06 + 0.0011$$
$$= \underline{\underline{4.5611}}$$

Lommeregner giver $f(2.02) = 4.56107679\ldots$

OPG. 2a)

$$\frac{dy}{dx} = -\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = -\frac{3x^2+6xy}{-4y^3+3x^2} = \underline{\underline{\frac{3x^2+6x\cancel{y}}{4y^3-3x^2}}}$$

$$\frac{dy}{dx} \text{ for } x=2 \text{ og } y=3 : \frac{3\cdot 4+6\cdot 2\cdot 3}{4\cdot 3^3-3\cdot 4} = \frac{12+36}{108-12} = \frac{48}{96} = \underline{\underline{\frac{1}{2}}}$$

OPG. 2b)

$$El_x(y) = \frac{x}{y}\cdot\frac{d\cancel{y}}{dx} = \underline{\underline{\frac{3x^3+6x^2\cancel{y}}{4y^4-3x^2y}}}$$

For $x=2$ og $y=3$ fås $El_x(y) = \frac{2}{3}\cdot\frac{1}{2} = \underline{\underline{\frac{1}{3}}}$

$x : 2 \to 2.12 \quad 6\%$

$y : 3 \to ? \quad 6\cdot\frac{1}{3}\% = \underline{\underline{2\%}}$

y vokser med ca. $6\cdot\frac{1}{3}\% = 2\%$, dvs.

y vokser fra 3 til ca. $\underline{\underline{3.06}}$

Alternativt: $y \approx 3+\frac{1}{2}\cdot 0.12 = 3.06$.

OPG. 3a)

$$\int(0.06x^2+0.5x)\,dx = \underline{\underline{0.02x^3+0.25x^2+c}}$$

$$\int_0^5(0.06x^2+0.5x)\,dx = \left[0.02x^3+0.25x^2\right]_0^5 = 2.5+6.25 = \underline{\underline{8.75}}$$

$$PS = 5\cdot 4-8.75 = 20-8.75 = \underline{\underline{11.25}}$$

OPG. 3b)

$$\int_a^5 2500x^{-4}\,dx = \left[2500\left(-\tfrac{1}{3}\right)x^{-3}\right]_a^5 = \underline{\underline{-\frac{20}{3}+\frac{2500}{3a^3}}}$$

$$CS = \lim_{a\to 0}\int_a^5 2500x^{-4}\,dx - 5\cdot 4 = -\frac{20}{3}+\infty-20 = \underline{\underline{\infty}}$$

$$CS_{ny} = \lim_{a\to 0}\int_a^5 \sqrt{80}\,x^{-\frac{1}{2}}\,dx - 5\cdot 4 = \left[\sqrt{80}\cdot 2\cdot x^{\frac{1}{2}}\right]_a^5 - 20$$

$$= \sqrt{80}\cdot 2\cdot\sqrt{5} - \lim_{a\to 0}\sqrt{80}\cdot 2\cdot\sqrt{a} - 20 = 40-20 = \underline{\underline{20}}$$

OPG. 4a) $\frac{\partial f}{\partial x} = 9x^2-9+y^2 \qquad \frac{\partial f}{\partial y} = 2xy$

Stat.pkt.: $9x^2-9+y^2=0$ og $2xy=0$

$2xy=0$ giver enten $x=0$ eller $y=0$.

$x=0$: $-9+y^2=0$, dvs $y=\pm 3$

$y=0$: $9x^2-9=0$, dvs $x=\pm 1$

Stat.pkt. er $(0,-3), (0,3), (-1,0)$ og $(1,0)$

OPG. 4b) $\frac{\partial^2 f}{\partial x^2} = 18x \quad \frac{\partial^2 f}{\partial x \partial y} = 2y \quad \frac{\partial^2 f}{\partial y^2} = 2x$

$(0,-3)$: $AC-B^2 = 0\cdot 0-(-6)^2 = -36$ sadelpkt.

$(0,+3)$: $AC-B^2 = 0\cdot 0 - 6^2 = -36$ sadelpkt

$(-1,0)$: $AC-B^2 = (-18)\cdot(-2)-0^2 = 36$ lok.max

$(1,0)$: $AC-B^2 = 18\cdot 2 - 0^2 = 36$ lok.min.

idet $AC-B^2<0$ giver sadelpkt.

$AC-B^2>0$ og $A<0$ giver lok.max

$AC-B^2>0$ og $A>0$ giver lok.min

OPG 5a)

$$T = \begin{pmatrix} 1 & 5 & 7 & 3 & a \\ 1 & 2 & 4 & 3 & a \\ 1 & 1 & 3 & 3 & 0 \end{pmatrix} \begin{matrix} -1 & -1 \\ & \\ & \end{matrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 5 & 7 & 3 & a \\ 0 & -3 & -3 & 0 & 0 \\ 0 & -4 & -4 & 0 & -a \end{pmatrix} \begin{matrix} & \\ -\frac{1}{3} & -\frac{4}{3} \\ & \end{matrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 5 & 7 & 3 & a \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -a \end{pmatrix} \begin{matrix} & \\ & \\ -1 & 1 \end{matrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 5 & 7 & 3 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a \end{pmatrix} \begin{matrix} \\ -5 \\ \end{matrix}$$

$$\downarrow$$

$$\begin{pmatrix} 1 & 0 & 2 & 3 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & a \end{pmatrix}$$

rang K = antal ledende 1 = 2

rang T = $\begin{cases} 2 \text{ for } a=0 \\ 3 \text{ for } a\neq 0 \end{cases}$

OPG 5b)

Ingen løsn. når $a \neq 0$
Uendelig mange løsn. når $a = 0$
Aldrig præcis én løsning.

For $a=0$ fås at x_1 og x_2 er basis-variable, mens x_3 og x_4 er frie variable.

Fuldstændig løsning $= \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2s-3t \\ -s \\ s \\ t \end{pmatrix} = \underline{\underline{\begin{pmatrix} -2 \\ -1 \\ 1 \\ 0 \end{pmatrix} s + \begin{pmatrix} -3 \\ 0 \\ 0 \\ 1 \end{pmatrix} t}}$

OPG 6a)

$$\det A = 0.8 \cdot 0.6 \cdot 0.8 + 0 + 0 - 0 - 0.2 \cdot 0.2 \cdot 0.8 - 0.2 \cdot 0.2 \cdot 0.8$$
$$= 0.384 - 0.032 - 0.032$$
$$= \underline{\underline{0.32}}$$

A^{-1} eksisterer da $\det A \neq 0$.

$$A^2 = \begin{pmatrix} 0.68 & 0.28 & 0.04 \\ 0.28 & 0.44 & 0.28 \\ 0.04 & 0.28 & 0.68 \end{pmatrix}$$

Plads 1,2 er udregnet sådan:

$$r_1 \cdot S_2 = (0.8,\ 0.2,\ 0) \cdot \begin{pmatrix} 0.2 \\ 0.6 \\ 0.2 \end{pmatrix} = 0.8 \cdot 0.2 + 0.2 \cdot 0.6 + 0 \cdot 0.2$$
$$= 0.16 + 0.12 = 0.28$$

De øvrige pladser tilsvarende!

OPG. 6 b)

$$A \cdot \begin{pmatrix} 0.5 \\ 0.2 \\ 0.3 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0.44 \\ 0.28 \\ 0.28 \end{pmatrix}}}$$

$$A^2 \cdot \begin{pmatrix} 0.5 \\ 0.2 \\ 0.3 \end{pmatrix} = A \cdot A \begin{pmatrix} 0.5 \\ 0.2 \\ 0.3 \end{pmatrix} = A \cdot \begin{pmatrix} 0.44 \\ 0.28 \\ 0.28 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0.408 \\ 0.312 \\ 0.280 \end{pmatrix}}}$$

Hvis fordelingen sidste år var 50%, 20%, 30% til V, D og S, så er fordelingen nu 44%, 28%, 28%, og næste år vil den være 40,8%, 31,2% og 28% (under forudsætning af samme overgangsmatrice)

OPG. 7a)

$$(I-A)\cdot(I-A)^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Af plads (1,1) og (1,2) fås:

$$1\cdot s + (-0.2)t + (-0.2)t = 1$$
$$1\cdot t + (-0.2)s + (-0.2)t = 0$$

dvs: $s - 0.4t = 1$ og $0.8t - 0.2s = 0$

Af den sidste ligns. fås $s = 4t$, og af den første ligns. fås så $3.6t = 1$, dvs.

$$t = \frac{1}{3.6} = \frac{10}{36} = \underline{\underline{\frac{5}{18}}} \quad \text{og} \quad s = 4t = \frac{20}{18} = \underline{\underline{\frac{10}{9}}}$$

OPG. 7b

$$(1500, 1500, 1500)\begin{pmatrix} 1 & -0.2 & -0.2 \\ -0.2 & 1 & -0.2 \\ -0.2 & -0.2 & 1 \end{pmatrix} = \underline{\underline{(900, 900, 900)}}$$

$$(p, q, r) = (u, v, w)\cdot\begin{pmatrix} s & t & t \\ t & s & t \\ t & t & s \end{pmatrix} = (990, 900, 900)\cdot\begin{pmatrix} s & t & t \\ t & s & t \\ t & t & s \end{pmatrix} =$$

$$(990s + 1800t,\ 900s + 1890t,\ 900s + 1890t) =$$

$$\left(990\cdot\frac{10}{9} + 1800\cdot\frac{5}{18},\ 900\cdot\frac{10}{9} + 1890\cdot\frac{5}{18},\ 900\cdot\frac{10}{9} + 1890\cdot\frac{5}{18}\right) =$$

$$\underline{\underline{(1600, 1525, 1525)}}$$

OPG. 8a)

$Av_1 = A\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 1\cdot v_1$, dvs v_1 egenvektor med $\underline{\underline{\lambda_1 = 1}}$

$Av_2 = A\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 0.8 \\ 0 \\ -0.8 \end{pmatrix} = 0.8v_2$, " v_2 " " $\underline{\underline{\lambda_2 = 0.8}}$

$Av_3 = A\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = \begin{pmatrix} 0.4 \\ -0.8 \\ 0.4 \end{pmatrix} = 0.4v_3$, " v_3 " " $\underline{\underline{\lambda_3 = 0.4}}$

OPG. 8 b)

$$Q = (x,y,z)A\begin{pmatrix}x\\y\\z\end{pmatrix} = (x,y,z)\cdot P \cdot P^{-1}A\cdot PP^{-1}\begin{pmatrix}x\\y\\z\end{pmatrix}$$

$$= (\mathbb{X},\mathbb{Y},\mathbb{Z})\cdot D\cdot\begin{pmatrix}\mathbb{X}\\\mathbb{Y}\\\mathbb{Z}\end{pmatrix} = \underline{\underline{\mathbb{X}^2+0.8\,\mathbb{Y}^2+0.4\,\mathbb{Z}^2}}$$

Da $\lambda_1>0, \lambda_2>0, \lambda_3>0$ er Q positiv definit (og altså også positiv semidefinit).

$$\underline{\underline{Q=0 \iff \mathbb{X}=\mathbb{Y}=\mathbb{Z}=0 \iff x=y=z=0}}$$

Korrekt besvarelse af mindst halvdelen af sættet sikrer beståelse.

Der gives ved bedømmelsen 0-10 points pr spørgsmål, dvs. 0-160 points i alt for de 16 spørgsmål.
Vejledende skema:

00-30	**points**	**giver -3.**
30-80	**points**	**giver 00.**
80-90	**points**	**giver 02.**
90-105	**points**	**giver 4.**
105-130	**points**	**giver 7.**
130-150	**points**	**giver 10.**
150-160	**points**	**giver 12.**