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GRAPH COLORING PROBLEMS
by
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Reviewer: Ron Read

At first sight this book would appear to be just another addition to the growing list of textbooks on specialized topics in Graph Theory; but closer scrutiny reveals that it is something quite different, and the authors are careful to point this out. Essentially, the book is a collection of graph coloring problems (over 200 of them) chosen to give the reader insight into the outstanding areas of current investigation and an idea of what are the most fertile fields for future research in this branch of mathematics. The problems have been chosen with certain criteria in view (for example, that they should be simple to state, and be specific rather than broad). Along with these problems the authors have included a wealth of background and history, so that the reader is well-informed of what is already known, as well as what is still to be discovered. For example, we have, on consecutive pages, Tait's theorem (1880) - that the faces of a 3-regular plane graph can be 4-colored if, and only if, the edges can be 3-colored - and Tutte's conjecture (1954) that every bridgeless graph has a 5-flow.

Although, for completeness, some problems, described as "impossible", have been included, most are of the kind where progress seems to be possible, and which may therefore be suitable as topics for research. The authors sum up the nature of the book when they say "We did not intend to write a textbook to be read from beginning to end, but rather a catalog suitable for browsing".

Unlike the prefaces to some books, the preface to this one should be read. It is by way of being an apologia by the authors, in which they justify their enthusiasm for graph coloring. Although the Four Color Conjecture has become the Four Color Theorem, graph coloring is very far from being a dead field. The chromatic number of a graph, for example, turns up in applications which are not directly concerned with the colorings of the graph; scheduling problems of many kinds are closely related to graph colorings, and so on. The most telling justification for the book, however, is that graph coloring is a subject which abounds in new and interesting problems.

A novel feature is the inclusion of instructions for sending or obtaining new and updated information concerning the problems in this book by anonymous ftp to Odense University. The authors also make known their intention to publish update articles from time to time.

Now that the general purpose and nature of the book has been described, what remains to be done is to give some indication of the contents of the several chapters. The first chapter is in a class by itself, being a comprehensive introduction to the theory of graph coloring, its history, nomenclature and fundamental results. Early in the chapter, after some basic definitions, comes the important link between coloring of infinite and finite graphs, as given by the theorem of De Bruijn and Erdös (1951) that if all finite subgraphs of an infinite graph can be colored in some finite number of colors then so can the infinite graph. Then comes a discussion of graphs on surfaces including, of course, the four color theorem. Then relations between chromatic number and maximum degree, and properties of critical graphs, sparse and random graphs, perfect graphs, edge colorings, integer flows and several generalizations of the concept of graph coloring are included.

Having given a general view of the territory to be explored, and having equipped readers for the expedition ahead, the authors offer the following invitation to the voyage: "So let us now embark on our journey into the jungle of these problems...." The remaining chapters in the book are the various stages of that journey. Here, however, the reviewer faces a problem. Each chapter is a treasure house of theorems, conjectures, open problems and historical comments. It would be quite impossible to say something about every topic treated. On the other hand, the range of subjects covered in each chapter is such that to single out one or two results for special mention would risk giving a wrong impression of the chapter as a whole. To escape from this dilemma I shall content myself with merely quoting the remaining chapter headings, since they will indicate, fairly accurately, the scope of the book. They are as follows:

- Chapter 2. Planar graphs.
- Chapter 3. Graphs on higher surfaces.
- Chapter 4. Degrees.
- Chapter 5. Critical graphs.
- Chapter 6. The conjectures of Hadwiger and Hajos.
- Chapter 7. Sparse graphs.
- Chapter 8. Perfect graphs.
- Chapter 9. Geometric and combinatorial graphs.
- Chapter 10. Algorithms.
- Chapter 11. Constructions.

- Chapter 12. Edge colorings.
- Chapter 13. Orientations and flows.
- Chapter 14. Chromatic polynomials.
- Chapter 15. Hypergraphs.
- Chapter 16. Infinite chromatic graphs.
- Chapter 17. Miscellaneous problems.

A comprehensive list of references is given for each chapter, and there is both an author index and a subject index at the end of the book.

All in all this is an excellent book which no one working in graph coloring, or having aspirations to do so, should be without.

Dr. Ronald C. Read is a professor in the Department of Combinatorics and Optimization at the University of Waterloo. His research interests lie in graph theory, especially the theory of chromatic polynomials, graphical enumeration and the application of computers to graph theory problems. His main hobby is music - both as a performer and a composer. Other hobbies include the study of string instruments, cryptography and (in earlier years) cave exploration.

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Editor's note: Please contact Ann Trenk (see email info on page 2) with your suggestions on books you would like to see reviewed or if you would like to volunteer to review a book.

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Software Exchange

Beginning with the next issue, the newsletter will have an area announcing software applications for discrete mathematics, both for research and education. If you'd like your program listed, please contact Jason Brown by email at brown@cs.dal.ca.