Exercises for Week 39

Parallel Computing, DM818 (Fall 2009)

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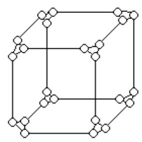
Exercise 1

Static Network Topology

We discussed the static network topology d-dimensional hypercube. A disadvantage of a hypercube-based architecture is that the degree grows with growing values for d. A modification of the hypercube is the so called cube-connected cycle graph (CCC-graph), that doesn't have that property.



Cube-connected cycle graph for $d = 3$

For a CCC-graph each node of a hypercube is replaced by a ring of d nodes. The set of nodes is defined by $V_{CCC} = V_H \times \{0, \ldots, d-1\}$. In a CCC-graph there is a edge between nodes (v, i) and (v', i') if

- $i = i'$ and v and v' differ only in the $i-$th bit, or
- $|i - i'| = 1$ and $v = v'$, or
- $i - i' = d$ and $v = v'$.

a) Determine the number of shortest paths between two nodes v_1 and v_2 in a standard hypercube. Hint: use the binary node labeling.

b) Determine the number of nodes and edges in a CCC-graph and of a hypercube with dimensionality d.

c) Determine the diameter of a CCC-graph for $d = 3$.

d) Determine the diameter of a CCC-graph for $d \geq 4$. Explain how you derived the formula and give an example for the case of $d = 4$. (A proof would be very complicated and is not necessary).

e) * (to be discussed in a later tutorial section). Explain an optimal routing algorithm in a CCC-graph. (The solution for this question is quite complicated, this part will not be presented in detail.)

Exercise 2

Hypercube

A cycle in a graph is defined as a path originating and terminating at the same node. The length of a cycle is the number of edges in the cycle. Show that there are no odd-length cycles in a d-dimensional hypercube.

Exercise 3

Hypercube

The *diameter* of a network is the maximum distance between any two processing nodes in the network. The *bisection width* of a network is defined as the minimum number of communication links that must be removed to partition the network into two equal halves. The *arc connectivity* of a network is the minimum number of edges that must fail (be removed from the network) to fragment the network into two unreachable parts. The *cost* of a network can be defined in many ways, for this exercise let the cost be the number of communication links between nodes.

Discuss all entries of Table 2.1 in the course book.

Exercise 4

Mesh of Trees

A mesh of trees is a network that imposes a tree interconnection on a grid of processing nodes. A $\sqrt{p} \times \sqrt{p}$ mesh of trees is constructed as follows. Starting with a $\sqrt{p} \times \sqrt{p}$ grid, a complete binary tree is imposed on each row of the grid. Then a complete binary tree is imposed on each column of the grid. Figure 1 illustrates the construction of a 4×4 mesh of trees. Assume that the nodes at intermediate levels are switching nodes. Determine the bisection width, diameter, and total number of switching nodes in a mesh of trees.

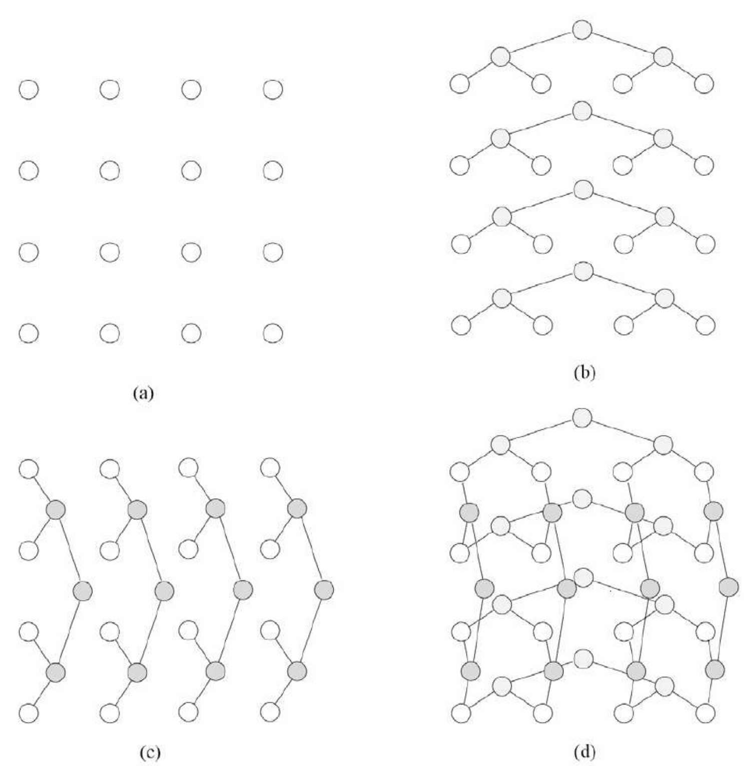


Figure 1: Mesh of Trees