

Figure 34.19 The reduction of 3-CNF-SAT to SUBSET-SUM. The form la in 3-CNF is $\phi=$ $C_{1} \wedge C_{2} \wedge C_{3} \wedge C_{4}$, where $C_{1}=\left(x_{1} \vee \neg x_{2} \vee \neg x_{3}\right), C_{2}=\left(\neg x_{1} \vee \neg x_{2} \vee \neg x_{3}\right), C_{3}=\left(\neg x_{1} \vee \neg x_{2} \vee x_{3}\right)$, and $C_{4}=\left(x_{1} \vee x_{2} \vee x_{3}\right)$. A satisfying assignment of $\phi$ is $\left\langle x_{1}=0, x_{2}=0, x_{3}=1\right\rangle$. The set $S$ produced by the reduction consists of the base- 10 numbers shown; reading from top to bottom, $S=\{1001001,1000110,100001,101110,10011,11100,1000,2000,100,200,10,20,1,2\}$. The target $t$ is 1114444 . The subset $S^{\prime} \subseteq S$ is lightly shaded, and it contains $v_{1}^{\prime}, v_{2}^{\prime}$, and $v_{3}$, corresponding to the satisfying assignment. It also contains slack variables $s_{1}, s_{1}^{\prime}, s_{2}^{\prime}, s_{3}, s_{4}$, and $s_{4}^{\prime}$ to achieve the target value of 4 in the digits labeled by $C_{1}$ through $C_{4}$.

