Institut for Matematik & Datalogi
Syddansk Universitet

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JBJ

DM201: Graph algorithms with applications — Weekly note 13

Final weekly note This is the last weekly note.

Oral exam It starts at 11.30 at June 9. There is 30 min preparation and we examine in the following order:

1. Kristian Kirknæs Færk: draw 11.30, exam at 12.00
2. Anders Sune Pedersen: draw 12.00, exam at 12.30
3. Kajetan Blazej Kubik: draw 12.30, exam at 13.00
4. Katarzyna Dabrowska: draw 13.00, exam at 13.30

You have approximately 20 minutes to cover the question that you draw and in the last 10 minutes we will ask questions in other parts of the pensum.

Pensum:

- J. Bang-Jensen and G. Gutin, Digraphs:Theory, Algorithms and Applications, Springer Verlag, London, 2001. pages 13-22, 28-38, 55-58, 82-85,95-170, 195-200, 234-238, 246-248, 250-253, 265-269, 345-367, 443-461, 500-502(middle), 549-553, 658-663.
- D. Jungnickel, Graphs, Networks and Algorithms, 3rd edition, Springer Verlag, Berlin, 2008. pages 13-20, 25-28, 72-76, 81-84, 127-143, 260-278, 387-449.
- B. Korte, J, Vygen, Combinatorial Optimization, Springer Verlag, Berlin, 2000. Pages 279-302.
- Hamiltonian paths in tournaments- a generalization of sorting, Notes by J. Bang-Jensen, 2006 (available from course page).
- The max-back ordering, Notes by Mette H. Eskesen, 2001 (available from course page).
- Optimum branchings, Section 9.2 in second ed. of Digraphs: Theory, Algorithms and Applications, Bang-Jensen and Gutin, 2008
- Hand written notes on the primal dual algorithm (12 pages)
- Chapter 5 “The primal-dual algorithm” from Papadimitriou and Steiglitz: Combinatorial Optimization, Prentice Hall, 1982.

Exam questions

1. Shortest and longest paths in digraphs with applications
   - Finding a negative cycle
   - Project scheduling
   - Train scheduling
2. Hamiltonian paths and cycles in tournaments and their generalizations.
   - Tournaments: fast algorithms and relation to sorting
   - Extended semicomplete and semicomplete bipartite digraphs (multi-insertion technique)
   - Quasi-transitive digraphs and the use of the structure theorem for these
   - Path-mergeable digraphs
3. Structure of generalizations of tournaments and the use in algorithms
   - Semicomplete multipartite digraphs
   - Quasi-transitive digraphs
   - Path-mergeable digraphs
4. Matroids and independence systems
   - Definitions
   - Greedy algorithm
   - Greedy algorithm as an approximation algorithm
5. Matroid intersection and matroid union
   - Definitions
   - Union of matroids is again a matroid
   - Applications (disjoint spanning trees, bipartite matching, ...)
   - Matroid intersection algorithm
6. Flows: concepts, max flow min cut theorem, min cost flows and algorithms
   - Residual network
   - Decomposition of flows
   - Reductions among flow models (lower bounds, circulations, balance vectors, (s,t)-flows, bounds on vertices).
   - MFMC theorem
   - Characterization of a minimum cost flow
   - Algorithms for minimum cost flows
7. Applications of flows
   - Matchings
   - Finding small cuts
   - scheduling
   - Path coverings
   - Chinese postman problem
8. Primal-dual algorithm
   - Description of the method, including combinatorial aspects
   - Application to the shortest path problem
   - Application to the assignment and the transportation problems
9. Connectivity
   - Determining connectivity via flows
   - Max-back orderings and edge-connectivity
   - Ear-decompositions

10. Connectivity augmentation
    - Problem description
    - Splitting off theorem(s)
    - Finding feasible splittings
    - Frank's algorithm
11. Branchings
    - Edmonds branching theorem
    - Minimum cost branchings
12. Matchings
    - Characterization of maximum matchings
    - Criterion for the existence of a perfect matching
    - Edmonds' blossom algorithm
    - Bipartite matchings via flows
    - Weighted matchings
13. Applications of matchings
    - Chinese postman problem
    - Shortest paths in undirected graphs
14. Orientations of graphs
    - Nash-Williams orientation theorem
    - Orientations with bounds on degrees
15. Submodular flows
    - Definitions and relation to standard flows
    - Feasibility theorem for fully submodular flows
    - Proof of Nash-Williams and Lucchesi-Younger theorems
    - Using submodular flows in algorithms for orientations