Institut for Matematik og Datalogi — October 3, 2023
Syddansk Universitet — JBJ

DM551/MM851 – Fall 2023 – Weekly Note 6

Stuff covered in week 40
We covered Kleinberg and Tardos 13.1-13.4.

NB: There are no Lectures in Weeks 41-43. There are exercises on October 9 and 13 and no other exercises in Weeks 41-44.

Instead you should spend the free time to do well on the first set of exam problems and watch two videos. Both are available under Course media gallery.

- Video on Chernoff bounds (Kleinberg-Tardos 13.9) to watch in Week 41 before second set of exercises.
- Video on Recurrence relations (Rosen 8.1-8.2) to watch by week 44.

Exercises in Week 41
Besides the ones below you can decide with Mads whether you want to discuss further problems from last years exam problems.

- Rosen page 591: 6,10,18 (note the we define D_0 to be 1)
- Exam January 2010, Problem 4.
- Exam January 2011, Problem 6
- Exam January 2015, Problem 4 and 6.
- Exam January 2014, Problem 3
- Kleinberg and Tardós Problem 1 page 782.

Connection between Stirling numbers $S(m,n)$ and the number of onto functions

Recall that the Stirling number $S(m,n)$ is the number of ways one can distribute m distinguishable elements into n non-distinguishable boxes so that every box will contain at least one element. We will show how to determine $S(m,n)$.
Consider the following two step construction of an onto function from a set X of m elements to a set $Y = \{y_1, y_2, \ldots, y_n\}$ of n elements. Think of another problem where we have n distinguishable boxes, labelled $1, 2, \ldots, n$. Now hide the labels of the boxes so that they look indistinguishable and do as follows.

1. Distribute the elements of X into the n boxes so that they are all non-empty.
2. Now reveal the labels of the boxes and obtain an onto function f from X to Y by mapping an element $x \in X$ to y_i if it was placed in the box whose label was i above.

This way, for each fixed way of performing step 1. there are $n!$ onto functions, namely the number of ways we could label the n boxes. This shows that the number of onto functions from a set on m elements to a set of n elements is precisely $n! \cdot S(m, n)$. Thus it follows from Theorem 1, page 588 that

$$S(m,n) = \frac{1}{n!}[n^m - \binom{n}{1}(n-1)^m + \binom{n}{2}(n-2)^m - \ldots + (-1)^{n-1}\binom{n}{n-1}]$$