Institut for Matematik og Datalogi
Syddansk Universitet, Odense

March 13, 2024
JBJ

DM553/MM850 – 2. Exam assignment

Hand in by friday April 5, 12:00.

Rules

This is the second of three sets of problems which together with the oral exam in June constitute the exam in DM553/MM850. **This set of problems must be solved alone!**. Any collaboration between students will be considered as exam fraud. Thus you may not discuss the solutions with other students. On the other hand, you can learn a lot from discussing the problems with each other so you may do this to some extend, such as which methods can be used or similar problems from the book or exercise classes.

It is important that you **argue so that the reader can follow your explanations**. It is not enough just to say that the solution follows from an example in the book or similar. In such a case you should explain **how/why** it follows from that example or result.

Remember that this (and the other two sets of assignments) counts as part of your exam, so do a good job and try to answer all questions carefully.

How to hand in your report

Your report must be handed in via Itslearning by Friday April 5 at 12:00

On the first page you must write your **name** and the first 6 digits of your **CPR-number**. Do not write the last 4 digits!

Exam problems

Solve the following problems. **Remember to justify all answers.**

PROBLEM 1 (30%)

Recall that a Turing machine M calculates a function $f : \Sigma^* \rightarrow \Sigma^*$ if, when starting in configuration $q_0 w$ (that is, with w on the input tape) M always halts and does so in the configuration $q_{accept} f(w)$.

Give short descriptions in words (do **not** use diagrams) of Turing machines M_1, M_2, M_3 for calculating the functions f_1, f_2, f_3, where

- $f_1(w) = (ww)^R$ (the parentheses are not part of the output, so the function produces the reverse of the string ww)
- $f_2(w) = w^{\lfloor |w|/2 \rfloor}$

- $f_3(w, z)$ is the function which takes the value $u\#v$ if there are strings $u, v \in \Sigma^*$ such that $w = uzv$ and otherwise $f_3(w, z)$ takes the value ϵ.

Your description must contain the most important steps of your algorithm when it is started on a legal input. You may assume that Σ is the universal alphabet (Video 10).

You may use a deterministic or a non-deterministic Turing machine and several tapes as you wish.

PROBLEM 2 (40%)

For each of the following claims you should say whether they are true or false AND you must give give a short explanation for your conclusion. In all statements concerning two languages you may assume that the two languages are over the same alphabet.

You will not get full credit for a correct but unjustified answer.

1. If A is a context-free language and B is a regular language, then the language $A \setminus B$ is context-free.
2. If A is a context-free language and B is a decidable language, then $A \cap B$ is decidable.
3. No context-free language can contain an undecidable language as a subset.
4. Every context-free language is generated by infinitely many different context-free gramars.
5. If A is decidable and B is decidable, then $A \cap B$ is decidable.
6. If A is undecidable and B is a finite language, then $A \cap B$ is decidable.
7. If A is a context-free language and B is a regular language, then $A \cap B$ is regular.
8. If A is decidable, then A is decided by infinitely many different Turing-machines.
9. If a language is finite, then it is decidable.
10. If A is undecidable and B is undecidable, then $A \cap B$ is undecidable.
11. The class of undecidable languages is closed under union, that is, if A and B are undecidable languages, then $A \cup B$ is also undecidable.
12. The class of Turing-recognizable languages is closed under union, that is, if L_1, L_2 are Turing-recognizable languages, then $L_1 \cup L_2$ is also a Turing-recognizable language.
13. The complexity class $\mathcal{NP}$ is closed under union and intersection.
14. Every language in $\mathcal{NP}$ is decidable.
15. If A and B are languages in $\mathcal{P}$, then $A \setminus B$ is also in $\mathcal{P}$.

16. If $A \setminus B$ is in $\mathcal{P}$ for every pair of languages A, B where in $A \in \mathcal{NP}$ and $B \in \mathcal{P}$, then $\mathcal{P} = \mathcal{NP}$.
17. The languages in $\mathcal{NP}$ can be enumerated $L_1, L_2, \ldots, L_i, L_{i+1}, \ldots$ (so $\mathcal{NP}$ contains only countably many languages).
18. There are uncountably many languages in $\mathcal{P}$.
19. If L can be enumerated by some enumerator E_L and $\bar{L}$ can be enumerated by some enumerator $E_{\bar{L}}$, then L is decidable.
20. Every language L in $\mathcal{NP}$ can be enumerated by some enumerator M_L.

PROBLEM 3 30%)

For each of the following languages you must say whether it is decidable or undecidable and give a short (but sufficient) explanation why this is so. You may use Rice's theorem when this applies. If you use it, you must argue why you can do so for the language in question. For each language which is decidable you must give a short explanation (a few lines in words) of how it can be decided by a deterministic Turing machine.

Below the strings $< M >, < G >, < w >$ always denote the universal encoding of a Turing machine M, a context-free grammar G and a string w over the relevant alphabet, respectively. Recall that the relevant alphabets are always finite sets.

You will not get full credit for a correct but unjustified answer.

1. $L_1 = \{< M > | M$ is a TM and there exists an NFA M' such that $L(M) = L(M')\}$
2. $L_2 = \{< M > | M$ is a TM and $L(M) \neq \emptyset\}$
3. $L_3 = \{< M >< w > | M$ is a non-deterministic TM that takes at most 110 steps on input $w\}$
4. $L_4 = \{< G >< M > | G$ is a CFG and M is a DFA and $L(G) \cap L(M) = \emptyset\}$
5. $L_5 = \{< M_1 >< M_2 > | M_1, M_2$ are Turing machines and $L(M_1) \subseteq L(M_2)\}$.
6. $L_6 = \{< G >< k > | G$ is a context-free grammar and $L(G)$ contains no string of length $k\}$.
7. $L_7 = \{< M >< w > | M$ is a TM and it visits some state at least 3 times when started on $w\}$.
8. $L_8 = \{< M > | M$ is a TM and $L(M) = L(M)^R\}$
9. $L_9 = \{< w > | w \neq w^R\}$
10. $L_{10} = \{< M >< w > | M$ is a PDA and $w \in L(M)\}$.