# Eksamensopgaver i DM17, Januar 2002

## Problem No. 1 (20 points)

Find a regular expression representing the following languages:

a) $L := \{w \in \{a,b\}^* \mid w$ has an odd number of occurences of $a\}$.

b) $L := \{w \in \{a,b\}^* \mid |w| \geq 3$ and the third symbol from the right is the letter $b\}$.

Consider the regular language $L$ over $\{0,1\}$ which is represented by the regular expression $(0 \cup 1)^*0$ .

c) Draw the state diagram of a nondeterministic finite automaton which precisely accepts $L$. The automaton should **not** be a deterministic one.

d) Draw the state diagram of a deterministic finite automaton which precisely accepts $L$.

## **Problem No. 2** (10 points)

Let $\Sigma$ be a finite alphabet and let $L \subseteq \Sigma^+$ be a regular language. Define a new language $\tilde{L}$ as the language of all words $w \in \Sigma^*$ such that $w$ is obtained from a word in $L$ by deleting its first letter.
Is the language $\tilde{L}$ regular as well? Prove your answer.

## Problem No. 3 (20 points)

Let the language $L \subset \{0,1\}^*$ be defined as

$$L := \{01^j \mid j \in \mathbb{N} \text{ such that there is a } k \in \mathbb{N} \text{ with } j = k! \} .$$

For example, the words 01 , 011 , 0111111 etc. are in $L$.

We want to prove that $L$ is not regular.

a) Show that using the Pumping Lemma the question whether $L$ is not regular can be reduced to the following problem:

   for any two natural numbers $a, b \in \mathbb{N}$ **not all** the numbers of the form

   $$a + n \cdot b \ , \ n \in \mathbb{N}$$

   are factorials of a natural number.

b) Prove that a) is true, i.e. given any two natural numbers $a, b \in \mathbb{N}$ show that there is a choice for $n \in \mathbb{N}$ such that $a + n \cdot b$ is not of the form $k!$ , no matter how we choose $k \in \mathbb{N}$.

**Problem No. 4** (20 points)

a) Consider the context-free grammar $G = (V, \Sigma, S, R)$, given by $V := \{S, A, B, a, b\}$ , $\Sigma := \{a, b\}$ and

$$\begin{aligned} R := \ & \{S \to AB \ , \ S \to Ba \ , \\ & A \to a \ , \ A \to abAS \ , \ A \to bAa \ , \\ & B \to b \ , \ B \to bSS \ , \ B \to aSBB\} \end{aligned}$$

Prove that all words in $L(G)$ contain the same number of occurences of the letters $a$ and $b$.

b) Consider the context-free language

$$L := \{a^n b^n \mid n \geq 1\} \cup \{a^n b^{2n} \mid n \geq 1\}$$

over $\Sigma = \{a, b\}$.
Write down a Pushdown automaton $M$ which precisely accepts $L$. This includes the definition of a transition table. Explain the way your PDA works.

**Hint for b):** To simplify the task your PDA is allowed to write in a single step several symbols onto the stack (but it can read only one at a time).

## **Problem No. 5** (15 points)

A function $f : \{a, b\}^+ \rightarrow \{a, b\}^+$ is given as follows: For input $x \in \{a, b\}^+$ we define $f(x)$ as the word which is obtained by replacing every occurence of the substring $ab$ in $x$ by the string $aa$.
For example:

$$\begin{aligned} f(aab) &= aaa \\ f(abbabb) &= aababb \\ &\vdots \end{aligned}$$

Give the full description of a Turing machine which for every input $x \in \{a, b\}^+$ replaces $x$ by $f(x)$ and stops (in a final state) on the right-most symbol of $f(x)$.
Explain your solution, i.e. explain the way your machine works.

## Problem No. 6 (15 points)

Let $\Sigma$ be a fixed finite alphabet with $|\Sigma| \geq 2$.
For each of the following problems over $\Sigma$ find out, whether they are decidable or not. You have to argue precisely, why your answer is correct.

a) INPUT: the encoding of a Turing machine $M$

QUESTION: does $M$ accept at least one word of length 1?

b) INPUT: the encoding of a Turing machine $M$

QUESTION: does $M$ have a transition rule whose application does not change the state?

c) INPUT: the encoding of a Turing machine $M$

QUESTION: is the language accepted by $M$ the complement of the halting problem, i.e. does $L(M) = \overline{HALT}$ hold?