Institut for Matematik og Datalogi
Syddansk Universitet

April 18, 2024
JBJ

DM553/MM850 – Spring 2024 – Weekly Note 11

Stuff covered in Week 16

- I showed that the greedy set covering heuristic has an approximation guarantee of $ln n$ and also mentioned that this is the assymptotically best approximation guarantee one can get unless P=NP.
- I covered the randomized $\frac{8}{7}$-approximation for Max-3-SAT and the LP-based 2-approximation algorithm for vertex cover. This is covered in Video 20
- I presented the fully polynomial approximation scheme for the subset sum problem from Cormen 35.5. This is covered in Video 21
- Algorithms and lower bounds for finding max and min as well as the two largest elements in a set of numbers using only comparisons. This is based on the notes from the home page: Baase pages 125-133 and my own notes pages 1-8. Both are available from the home page of the course. Videos 22A and 22B.

Key points

- We saw many different ways of developing approximation algorithms: greedy approaches for vertex cover and for set cover, randomized algorithm for MAX-3-SAT and the linear programming based 2-approximation algorithm for weighted vertex cover.
- Subset sum is easily solved in exponential time by just, successively and for each $i = 1, \ldots, n$, generating all possible integers that are at most the target t and can be obtained as a sum of the first i integers in the list (of size n). Since the $(i+1)$st list may contain twice as many numbers as the ith this may take exponential time when t is large AND the numbers are not bounded by a function of n.
- In order to obtain a polynomial approximation scheme we **trim** the ith list before proceeding to generate the $(i+1)$st list. This is done by moving through the $(i+1)$ untrimmed sorted list from left to right and deleting the next number if it is within the trimming distance from the largest element currently kept in the list (so the value 0 (corresponding to picking the empty sum) and the smallest element from the input is always kept in the list).

New material in Week 17
This is covered by Video 23

- I will probably recall the lower bound argument for minimum and second smallest argument.
- Algorithms and lower bounds for finding the median of n numbers. This is based the notes from Baase and my own notes.

Exercises in Week 17:

- Cormen 35.1-5
- Cormen 35.2-3 Hint: Let H_1 be any vertex and for $i = 2, \ldots, n$ let $u_i v_i$ be the shortest edge between $V(H_{i-1})$ and $V \setminus V(H_i)$ with $u_i \in V(H_{i-1}$. Show by induction that in step i (adding the i'th vertex) the cost of the current tour H_i is at most 2 times the sum $\sum_{j=2}^{i} c(u_j v_j)$. Now compare with the way Prim's algorithm for minimum spanning tree works to see that the cost of the final H_n is at most twice the cost of a minimum spanning tree.
- Cormen 35.3-4
- Cormen 35.4-3
- Cormen 35.4.
- JBJ notes exercises 1,2,4,6,8