Institut for Matematik og Datalogi
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# DM553/MM850 – Spring 2024– Weekly Note 3

**Material covered in week 6, 2024:**
We covered Sections 2.1, 2.2 and Section 2.3 (Videos 3,4,5,6A,6B)
Key points:

- A pushdown automaton (PDA) is a nondeterministic finite automaton enhanced with a stack which makes PDA's more powerful than DFA's and NFA's.
- If we require that a PDA is deterministic, then we loose power in terms of which languages can be recognized. Note that this did not happen for Finite automata!
- I mentioned the following important fact without proof (see the paper listed under notes on bottom of the homepage) that **every context-free language over a one-symbol alphabet is also regular**. You are not supposed to be able to prove this, but you must be able to use this fact.
- There exist languages that are not context-free, and the pumping lemma can be used to prove that this is the case for a given language. As for regular languages, the proof goes by contradiction. A typical example of a non-context free language is $\{a^n b^n c^n | n \geq 0\}$.
- Every regular language is context free.
- The class of context free languages is not closed under intersection. Note that this just means that there exists context free languages $L_1, L_2$ such that $L_1 \cap L_2$ is not context free. If we take $L_1 = \{a^i b^j c^k | i, j, k \geq 0 \text{ and } i = j\}$ and $L_2 = \{a^i b^j c^k | i, j, k \geq 0 \text{ and } j = k\}$, then both of these are context free but $L_1 \cap L_2 = \{a^n b^n c^n | n \geq 0\}$ which we know is not context free.
- PDA's are equivalent to context-free grammars, since, for any PDA A, one can construct a grammar G such that L(A) = L(G), and vice versa.
- The class of context-free languages is closed under union, concatenation, and star, but **not** under intersection and complement. Note that this is NOT the same as saying that the complement of a context-free language is never context free. For example both $\Sigma^*$ and its complement, the empty set are context-free (they are also regular). Also $L = \{a^n b^n | n \geq 0\}$ and its complement are context-free (but not regular).
- The intersection of a context free language $L_1$ with a regular language $L_2$ is again a context-free language. This can be seen by observing that if we are given a PDA $M_1$ for the context free language $L_1$ over $\Sigma$ and a DFA $M_2$ for the regular language $L_2$, then we can make a PDA $M$ which simulates $M_1$ and $M_2$ in parallel: think of having pairs of states $(q, p)$ where $q$ is the current state of $M_1$ and $p$ the current state of $M_2$. Then on a symbol $a \in \Sigma$ the PDA $M$ will behave as $M_1$ in the first coordinate (including updating the stack as $M_1$ would) and as $M_2$ in the second coordinate. Now we can make $M$ accept a string $w$ precisely when $M_1$ would accept $w$ (be in an accepting state with empty stack AND $M_2$ would be in an accepting state after reading $w$. Thus $M$ accepts $w$ if and only if $w \in L_1 \cap L_2$, so $L_1 \cap L_2$ is context free (as it is accepted by a PDA).

**New material to be covered in Week 7**

- Section 3.1 on Turing Machines (Video 7).
- I may also say a bit about extensions of Turing machines. This will be covered carefully in Week 8.

**Exercises (remember there are two sets of exercise classes) :**

**Numbers and pages are listed according to the 3rd international edition (mine has print year 2016).** In parenthesis are numbers and pages in the new 3rd edition that some of you may have (in the copy I have it says print year 2021 and the frontpage is black with an old airplane on it).
The exams refered to below are previous written exams in former courses on computability. They are available from the home page. Unfortunately several of them are only in danish, but as there are very few words you will be able to see what is asked for.

- 1.51 (a),(c),(d) on page 91 (1.46 (a),(c),(d) page 90).
- 2.2 on page 154 ( 2.2 on page 154 ).
- 2.9 on page 155, except the question on ambiguity (2.9 on page 155),
- 2.32 on page 158 (2.20 page 156).
- 2.42 on page 158 (2.30 page 157).
- 2.44 on page 158 (2.32 page 157).
- 2.58 on page 160 (2.47 page 159).
- Show that the class of context-free languages is closed under union, that is, if $L, L'$ are context-free languages over the same alphabet $\Sigma$, then $L \cup L'$ is also context-free.
- Problem 4, January 2002.
- Problem 4, January 2007.
- Problem 2, January 2008.
- Problem 2, October 2010.