Institut for Matematik og Datalogi
Syddansk Universitet

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JBJ

DM553/MM850 – Spring 2024 – Weekly Note 6

Stuff covered in week 9

- Sipser Chapter 4
- Sipser 5.1 pages 215-220 (in both books). The rest of Section 5.1 as well as Section 5.2 will not be covered and are not part of the pensum for the course.
- Sipser 5.3

Key points

- Many problems concerning regular and context-free languages are decidable.
- A problem/language is said to be **undecidable** if no TM decides it.
- There are only countably many distinct Turing machines and we can list their codes lexicographically (over the universal alphabet) as $< M_1 >, < M_2 >, \ldots, < M_i >, \ldots$.
- As a consequence there are only countably many decidable languages, because each TM which is a decider decides precisely one language, namely $L(M)$.
- The language $A_{TM} = \{< M >< w > | M \text{ is a TM and } w \in L(M)\}$ is undecidable. We proved this by using a hypothetical TM H which decides A_{TM} to construct a TM D that is not in the list of all Turing machines (because it disagrees the the i'th TM M_i on the string $< M_i >$). This contradiction shows that D and hence H cannot exist (D is a simple modification of H so if H existed, then also D would exist).
- Another example of an undecidable problem is the **Halting problem** which asks whether a program halts on a certain input. The Halting problem is recognizable, though since the universal Turing machine can be used to simulate a given TM on a given input and accept exactly those pairs $< M, w >$ for which M halts on w.
- A **mapping reduction** from a Language A to a language B is a computable function f so that $f(x)$ is in B if and only if x is in A. We denote this by $A \leq_M B$.
- If $A \leq_M B$ and A is undecidable, then B is also undecidable.
- If $A \leq_M B$ and B is recognizable, then A is also recognizable. the languages A_{TM} and E_{TM}.

New material in Week 10 (Video 14)

- More undecidable problems from Section 5.1
- Rice's theorem (Sipser Exercise 5.16, I will give a detailed coverage)

Exercises in Week 10

- 4.2, 4.3, 4.4 (same numbers in 3rd edition)
- 4.11 (4.31 in 3rd ed.)
- 4.20 (4.21 in 3rd ed.)
- 4.25 (4.17 in 3rd ed.). Hint: show that length mn works where m is the number of states of A and n is the number of staes of B and A, B are the two given DFAs.
- 4.29 (4.13 in 3rd ed.)
- October 2010 problem 3.
- October 2010 problem 5.
- October 2011, Problem 4 (b)