Institut for Matematik og Datalogi
Syddansk Universitet

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JBJ

DM553/MM850 – Spring 2024 – Weekly Note 8

Stuff covered in Week 11

- Sections 7.2 and 7.3 on the complexity classes P and NP (Video 15).
- Cormen Section 34.2 (read this yourselves. Most of it is covered also in Sipser).
- Sipser 7.4 untill the Cook-Levin Theorem. (Video 16)
- Cormen 34.3 untill the Section on circuit satisfiability.(the rest of Section 34.3 and Cormen Section 34.4 is not pensum!) (Video 16)

Key points

- The complexity class P is the set of all languages that can be decided in polynomial time by a deterministic Turing machine.
- The complexity class NP is the set of all languages that have polynomial time verifiers. This is the same as the set of all languages that can be decided in polynomial time by a nondeterministic Turing machine.
- Every decidable language L which is non-trivial (that is, $L \neq \emptyset$ and $L \neq \Sigma^*$) is **complete** for the classe of decidable languages under **mapping reductions**. To see this, we argue as follows. As L is non-trivial there are strings w_{in}, w_{out} such that $w_{in} \in L$ and $w_{out} \notin L$. Now let L^* be a language that is decided by a deterministic TM M^*. Then we can form a mapping reduction $L^* \leq_M L$ as follows: On input x first use M^* to decide whether $x \in L^*$. If $x \in L$ then we set $f(x) = w_{in}$ and otherwise we set $f(x) = w_{out}$. It is easy to check that this is a mapping reduction from L^* to L.
- Similarly we can show that every non-trivial language in $\mathcal{P}$ is complete for $\mathcal{P}$ under **polynomial** reductions.
- A Language L is **NP-complete** if $L \in NP$ and $L' \leq_P L$ for every language $L' \in NP$. The last part says that every other language in NP can be reduced to L in polynomial time, a seemingly impossible thing the prove (e.g. as there are infinitely many languages in NP). However, the Cook-Levin Theorem (which we will prove a bit later) establishes that Satisfiability is NP-complete by using the definition of NP as the class of languages that are decidable in polynomial time by some NDTM to establish such a proof.

New material in Week 12:
Most of this is covered in Video 17.

- Cormen 3rd edition Section 34.5.
- Sipser 7.5 pages 311-314. The rest of the section is not pensum.

Exercises in week 12:

- Cormen 34.2-7, 34.2-9, 34.2-10
- Cormen 34.2-11. Show by an example that the square G^2 of G (where two vertices are connected by an edge if their distance in G is at most 2) does not have to be hamiltonian. It is a non-trivial result that if $G - v$ is connected for every vertex of G, then G^2 is in fact hamiltonian.
- Cormen 34.5-1,34.5-2, 34.5-3, 34.5-6, 34.5-7 (same numbers in 4th edition)