Institut for Matematik og Datalogi
Syddansk Universitet

March 31, 2024
JBJ

DM553/MM850 – Spring 2024 – Weekly Note 9

Obligatory assignment 2

Remember that the handin date is April 5 at 12.00.

Stuff covered in Week 12, 2024
This corresponds to Video 17

- Proof that Vertex cover are NP-complete (from Sipser)
- Proof that Subset-sum is NP-complete (based on Cormen 34.5)
- The proof from Cormen that HAMCYCLE is NP-complete. Note that I changed the slides slightly to make it easier to see the connection between the Hamiltonian cycles and vertex covers.
- The proof that TSP is NP-complete

Key points

- A Language L is **NP-complete** if $L \in NP$ and $L' \leq_P L$ for every language $L' \in NP$. The last part says that every other language in NP can be reduced to L in polynomial time, a seemingly impossible thing the prove (e.g. as there are infinitely many languages in NP). However, the Cook levin Theorem (which we will prove after easter) establishes that Satisfiability is NP-complete by using the definition of NP as the class of languages that are verifiable by some NDTM to establish such a proof.
- I gave several NP-completeness proofs from Sipser and Cormen, the most difficult one being that HAMCYCLE is NP-complete (by showing a polynomial non-trivial reduction from Vertex cover to HAMCYCLE).
- I also showed that TSP is NP-complete by providing an easy (polynomial) reduction from HAMCYCLE to TSP.

No teaching in Week 14
Use the extra time to make sure your solutions to the second set of exam problems are as good as possible.

New material in Week 15, 2024

- Approximation algorithms part 1 (Video 19) Cormen 35.1-35.2. We start by covering this on April 8.
- The Cook-Levin Theorem (Video 18).

Exercises in Week 15:

All Cormen references are identical for editions 3 and 4.

- Cormen 34.5-3, 34.5-7
- Cormen 34.5-5. Hint: reduce the subset sum problem to the set partitioning problem.
- Cormen 34.5-6. Hint: make a small modification of the graph G' we used in the proof that HAMCYCLE is NPC.
- Cormen 34-1 Page 1101 (a)-(c).
- Cormen 34-3.
- Prove that TSP is NP-complete even if we insist that the edge costs satisfy the triangle in-equality, that is, if uv, vw and uw are edges then the cost of uw is at most the sum of the costs of uv and vw.
- Cormen 35.1-2, 35.1-4, 35.1-5.