Formulations of Integer Programs

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What is an integer program?

Let us start with a linear program:

$$\max\{cx | Ax \leq b, x \geq 0\}$$

where A is a m by n matrix, c is a vector of size n, b a vector of size m and x is a vector of (decision) variables.

Which is equivalent to writing:

$$
\begin{array}{lllllll}
\max & c_1x_1 & +c_2x_2 & \dots & +c_nx_n & & \\
\text{s.t.} & a_{11}x_1 & +a_{12}x_2 & \dots & +a_{1n}x_n & \leq & b_1 \\
 & a_{21}x_1 & +a_{22}x_2 & \dots & +a_{2n}x_n & \leq & b_2 \\
 & \vdots & \vdots & & \vdots & & \vdots \\
 & a_{m1}x_1 & +a_{m2}x_2 & \dots & +a_{mn}x_n & \leq & b_m \\
 & x_1 \geq 0, & x_2 \geq 0, & \dots & x_n \geq 0 & &
\end{array}
$$

Mixed Integer Program

Now if *some* but not all variable are integer, we have a **(linear) Mixed Integer Program (MIP)**:

$$\begin{array}{lllll} \max & cx & + & hy & \\ \text{s.t.} & Ax & + & Gy \leq b & \\ & x \geq 0, & & y \geq 0 \text{ and integer} & \end{array}$$

where A is a m by n matrix, G is a m by p matrix, c is a vector of size n, h is a vector of size h, b is a vector of size m and x is a vector of (decision) variables, and finaly y is a vector of **integer** (decision) variables.

Integer Program (IP)

If *all* variables are integer, we have a **(linear) Integer Program**:

$$\begin{array}{lrcl} \max & cx & & \\ \text{s.t} & Ax & \leq & b \\ & x & \geq & 0 \text{ and integer} \end{array}$$

Binary Integer Program (BIP)

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And if all variables are not only integer but restricted to the values 0 or 1, we have a **Binary Integer Program**:

$$\begin{array}{lll} \max & cx & \\ \text{s.t.} & Ax & \leq \quad b \\ & x & \in \quad \{0,1\}^n \end{array}$$

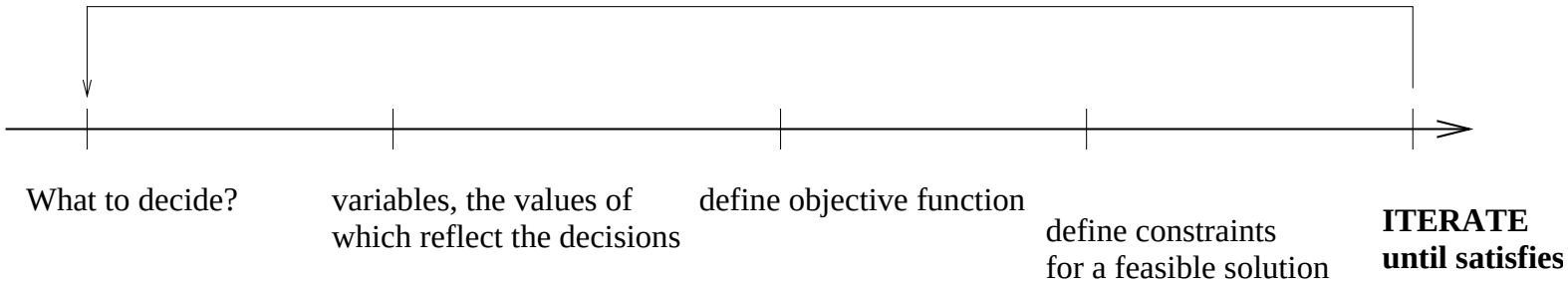
Combinatorial Optimization Problem

For the **Combinatorial Optimization Problem** we are given a finite set $N = \{1, 2, 3, \ldots, n\}$, weights c_j for each $j \in N$, and a set F of **feasible** subsets of N.

$$\max_{S \subset N} \{ \sum_{j \in S} c_j : S \in F \}$$

Different Models:

- How does one model?



Modelling is an **art**.

Formulating IPs and BIPs

- The assignment problem: assigning people to jobs.
- The Knapsack problem: Determine best collection.
- Set Covering: Who does which job.
- Travelling Salesman: Visiting customers/cities.
- Uncapacitated Facility Location: Locating depots and assigning customers.
- Uncapacitated Lot Sizing: Production planning.