Project Planning

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Project Management

Project Management is a set of techniques that helps management manage large-scale projects; projects that typically require coordination of numerous tasks throughout the organisation.

- PERT = Program Evaluation and Review Technique
- CPM = Critical Path Method

The methods were developed independently but are now often used interchangeably and are combined in one acronym: PERT/CPM.

Example

The Reliable Construction Company has just made the winning bid of a $ 5.4 million contract to construct a new plant.

There are the following constraints:

- a penalty of $ 300,000, if Reliable has not completed construction by the deadline of 47 weeks from now, and
- a bonus for speedy construction of $ 150,000 to be paid if the plant is ready within 40 weeks.

Questions

- How can the project be displayed graphically to better visualize the flow of the activities?
- What is the total time required to complete the project if no delays occur?
- When do the individual activities need to start and finish (at the latest) to meet this project completion time?
- When can the individual activities start and finish (at the earliest) if no delays occur?

More questions

- Which are the critical bottleneck activities where any delays must be avoided to prevent delaying project completion?
- For the other activities, how much delay can be tolerated without delaying the project completion?
- If extra money is spent to expedite the project, what is the least expensive way of attempting to meet the target completion time?

Project Networks

Project networks consists of a number of nodes and a number of arcs. There are two alternatives for presenting project networks.

- **Activity-on-arc (AOA):** Each activity is presented as a an *arc*. A node is used to separate an activity from each of its immediate predecessors.
- **Activity-on-node (AON):** Each activity is represented by a node. The arcs are used to show the precedence relationships.

AON vs. AOA

- AON are considerably easier to construct than AOA.
- AON are easier to understand than AOA for inexperienced users.
- AON are easier to revise that AOA when there are changes in the network.

Critical path

A path through a project network is one of the routes following the arcs from the START node to the FINISH node. The length of a path is the sum of the (estimated) durations of the activities on the path.

St->A->B->C->D->G->H->M->Fi	40
St->A->B->C->E->H->M->Fi	31
St->A->B->C->E->F->J->K->N->Fi	43
St->A->B->C->E->F->J->L->N->Fi	44
St->A->B->C->I->J->K->N->Fi	41
St->A->B->C->I->J->L->N->Fi	42

Scheduling individual activities

- The PERT/CPM scheduling procedure begins by addressing when the activities need to start and finish at the earliest if no delays occur?
- Having no delays mean 1) actual duration equals estimated duration and 2) each activity begins as soon as all its immediate predecessors are finished.
  - ES = Earliest start for a particular activity
  - EF = Earliest finish for a particular activity

where EF = ES + duration

Earliest Start Time Rule

The earliest start time of an activity is equal to the **largest** of the earliest finish times of its immediate predecessors, or

$$\text{ES} = \max \{ \text{EF of immediate predecessors} \}$$

Finding latest start and finish times

The **latest start time for an activity** is the latest possible time that it can start without delaying the completion of the project (so the finish node still is reached at its earliest finish time), assuming no subsequent delays in the project. The **latest finish time** has the corresponding definition with respect to finishing the activity.

- LS = latest start time for a particular activity
- LF = latest finish time for a particular activity

where LS = LF - duration

Latest Finish Time Rule

The latest finish time of an activity is equal to the **smallest** of the latest start times of its immediate successors, or

LF = $\min$ { LS of the immediate successors }

Identifying slack

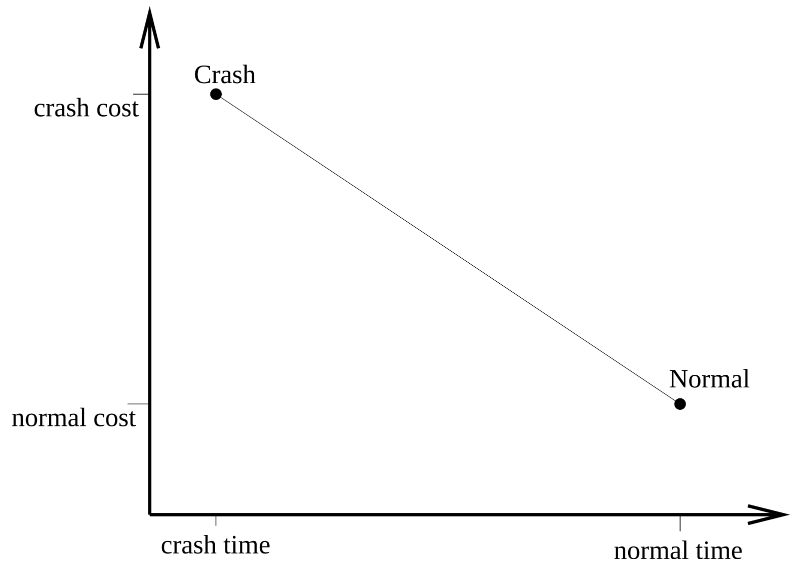
The **slack for an activity** is the difference between its latest finish time and its earliest finish time.

Slack	Activities
0	A, B, C, E, F, J, L, N
pos.	D, G, H, I, K, M

Each activity with zero slack is on a critical path through the project network such that any delay along this path will delay project completion.

Crashing an activity

Crashing an activity refers to taking (costly) measures to reduce the duration of an activity below its normal time. **Crashing the project** refers to crashing a number of activities in order to reduce the duration of the project below its normal value.



Considering Time-Cost trade-offs

What is the least expensive way of crashing some activities to reduce the estimated project duration to the specified level?

One way: look at marginal costs.

Using LP to make crashing decisions

Restatement of the problem: Let Z be the total cost of crashing activities. The problem then is to minimize Z, subject to the constraint that project duration must be less than or equal to the time desired by the project manager.

Modelling using LP

- What are our decision variables? x_j = reduction in the duration of activity j due to crashing this activity.
- How will our objective function look like? The objective function is to minimize the total cost of crashing activities:
  $\min 100,000x_A + 50,000x_B + \ldots + 60,000x_N$.
- To impose the constraint that the project must be finished in less than or equal to a certain number of weeks we impose another variable: y_{Fi}.

Modelling using LP (cont)

- To help the LP assign the appropriate value to y_F given the values of $x_A, x_B, \ldots$ it is convenient to introduce: y_j = start time of activity j.
- NOW since the start time of each activity is directly related to to the start time and duration of each of its immediate predecessors we get:

  start time of this activity $\geq$ (start time - duration) for this immediate predecessor

Putting it all together

1. $\min 100,000x_A + 50,000x_B + \ldots + 60,000x_N$
2. Maximum reduction constraints:
   $x_A \leq 1, x_B \leq 2, \ldots, x_N \leq 3$.
3. Non-negativity constraints:
   $x_j \geq 0, y_j \geq 0, y_{Fi} \geq 0$
4. Start time constraints: For each arc we get a relationship between the two end points:
   $y_C \geq y_B + 4 - x_B$, $y_D \geq y_C + 10 - x_C$ etc.
5. Project duration constraints: $y_{Fi} \leq 40$