Parameterized Complexity for Graph Linear Arrangement Problems

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A *linear arrangement (LA)* of a graph $G = (V, E)$ of order n is a bijection $\alpha : V \to \{1, \ldots, n\}$. The *length* (*net length*) of an edge $uv \in E$ relative to α is defined as $\lambda_\alpha(uv) = |\alpha(u) - \alpha(v)|$ ($\lambda_\alpha(uv) = |\alpha(u) - \alpha(v)| - 1$). The *cost* (*net cost*) of an LA α is is the sum of the costs (net costs) relative to α of edges of G. For an LA α of $G = (V, E)$, its *profile* is $\mathrm{prf}(G) = \sum_{v \in V} \max\{\alpha(v) - \alpha(u) : u \in N[v], \alpha(u) \leq \alpha(v)\}$.

In his recent Habilitation thesis and at his talk at Dagstuhl in July 2005, H. Fernau considers the following problem: given a graph $G = (V, E)$ and a parameter k, check whether there is an LA of net cost at most k. Fernau asks whether the problem is fixed-parameter tractable (FPT), i.e., whether it can be solved by an algorithm of time complexity $O(f(k)(|V| + |E|)^t)$, where t is a constant independent of k and f is a computable function. We prove that the problem is FPT by deriving an algorithm of complexity $O(|V| + |E| + 5.88^k)$.

M. Serna and D.M. Thilikos (2005) ask whether the following problems are FPT: (i) given a graph $G = (V, E)$ and a parameter k, check whether there is an LA of cost at most $k|V|$; (ii) given a graph $G = (V, E)$ and a parameter k, check whether there is an LA of cost at most $k|E|$; (iii) given a graph $G = (V, E)$ and a parameter k, check whether there is an LA of profile at most $k|V|$. We prove that for any fixed $k \geq 2$ the following problems are NP-complete: check whether $G = (V, E)$ has an LA of cost at most $k|V|$ (cost at most $k|E|$, profile at most $k|V|$). Thus, the Serna-Thilikos problems are not FPT.