Design of fault tolerant on-board networks

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Modern telecommunications satellites are very complex to design and an important industrial issue is to provide robustness at the lowest possible cost. A key component of telecommunication satellites is an interconnection network which allows to redirect signals received by the satellite to a set of amplifiers where the signals will be retransmitted. In this paper, we consider a certain type of interconnection network as asked by Alcatel Space Industries. The network is made of expensive switches ; so we want to minimize their number subject to the following conditions : Each input and output is adjacent to exactly one link ; each switch is adjacent to exactly four links ; there are n inputs (signals) and $n + f$ outputs (amplifiers) ; among the $n + f$ outputs, f can fail permanently; among the n input signals, p of them called priorities must be connected to the amplifiers providing the best quality of service (that is to some specific outputs) and the other signals should be sent to other amplifiers. Note that the priority signals are given, but the amplifiers providing the quality of service change during the life of the satellite and so the networks should be able to route the signals for any set of f failed outputs and any set of p best quality outputs.

This problem can be formally restated as follows: An $(n, p, n + f)$-*network* G is a graph (V, E) where the vertex set V is partitioned into four subsets $\mathcal{P}$, $\mathcal{I}$, $\mathcal{O}$ and $\mathcal{S}$ called respectively the *priorities*, the *ordinary inputs*, the *outputs* and the *switches*, satisfying the following constraints:

- there are p priorities, $n - p$ ordinary inputs and $n + f$ outputs;
- each priority, each ordinary input and each output is connected to exactly one switch;
- switches have degree at most 4.

An $(n, p, n + f)$-network is a (n, p, f)-*repartitor* if for any disjoint subsets $\mathcal{F}$ and $\mathcal{B}$ of $\mathcal{O}$ with $|\mathcal{F}| = f$ and $|\mathcal{B}| = p$, there exist in G, n edge-disjoint paths, p of them from $\mathcal{P}$ to $\mathcal{B}$ and the $n - p$ others joining $\mathcal{I}$ to $\mathcal{O} \setminus (\mathcal{B} \cup \mathcal{F})$. The set $\mathcal{F}$ corresponds to set of failures and $\mathcal{B}$ to the set of amplifiers providing the best quality of service. We denote $R(n, p, f)$ the minimum number of switches (i.e. cardinality of $\mathcal{S}$) of a valid (n, p, f)-repartitor. A (n, p, f)-repartitor with $R(n, p, f)$ switches will be called *minimum*.

The goal is to determine $R(n, p, f)$ and construct a minimum (or almost minimum) repartitor.

After giving some explicit constructions of minimum (n, p, f)-repartirors for small values of p anf f, we show a linear upper bound on $R(n, p, f)$: $R(, n, p, f) \leq 34(n+p+f)$.